Intertemporal Product Management with Strategic Consumers: The Value of Defective Product Returns

Abstract

(1) **Problem Definition:** An increased incidence of quality issues, resulting in defective product returns (DPRs), is a concern for firms bringing innovative products to market. While a firm can recover value from DPRs through refurbishing, consumers are known to act strategically in anticipation of the future availability of refurbished units. We study a firm’s strategy for offering a new product and refurbished DPRs to strategic consumers across time.

(2) **Academic/Practical Relevance:** Aided by emerging shopping tools, an increasing number of consumers consider buying refurbished versions of products rather than their new counterparts. A novel contribution of our work is the recognition of the refurbishing of DPRs as a possible solution to the time inconsistency problem that arises when a firm offers products to strategic consumers across time. We characterize how the product line decisions and profit of the firm are influenced by the defect rate, the perceived quality of refurbished DPRs, and consumers’ hassle cost of returns.

(3) **Methodology:** We develop a two-period game-theoretic model to characterize the firm offering the new product and refurbished DPRs to strategic consumers across time.

(4) **Results:** The refurbishing of DPRs helps the firm implicitly commit to limiting the quantity of the new product offered in the future, allowing the firm to charge a premium for the new product today. As a result, firm profit may even increase with the defect rate. These results persist across various model extensions.

(5) **Managerial Implications:** Whereas the firm’s profit is the highest when there are no defects, opportunities to achieve marginal reductions in defect rates may not be worth the investment, and may even be counterproductive. Also, efforts towards enhancing the perceived quality of the refurbished product or decreasing the hassle cost for consumers may better serve the firm than efforts towards marginally improving defect rates.

*Key words: defective product returns (DPRs); refurbishing; strategic consumers; time inconsistency*
1. Introduction

An increased incidence of quality issues, resulting in defective product returns (DPRs), is a major concern for firms bringing innovative products to market (Blumfield 2016, Calantone and di Benedetto 1988, Schneider and Hall 2011). Recent examples include: Samsung’s flagship smartphone – the Galaxy Note 7, where the quality problem was narrowed down to battery manufacturing issues at two suppliers (Chen and Sang-Hun 2016, Heathman 2017); Nintendo’s Switch gaming system, where a manufacturing variation was identified as the source of interruptions in the wireless signal in a subset of controllers (Coldewey 2017); and, a variety of quality issues that have affected components in some of Apple’s products, such as the iSight camera in certain iPhone 6 Plus units, video components in certain MacBook Pro systems, and the sleep/wake button in certain iPhone 5 units (Apple 2017). More broadly, according to a Consumer Electronics Association (CEA, now the Consumer Technology Association) study, 22% of consumers returned a newly purchased CE device in 2014, and 62% of consumers who returned a CE device in the two years preceding the study claimed the device was defective in some way (Koenig 2014). Furthermore, according to a 2011 CEA study, when returning a CE device, the most common action by consumers is exchanging the device for an exact replacement (same model and brand), whereas the second-most common action is a refund request (Business Wire 2011).

While firms justifiably perceive DPRs as a costly component of doing business (Petersen and Kumar 2009, Stock et al. 2006), value from DPRs can be recovered through refurbishing (see Lee and Kim 2016, Orland 2013, and Broida 2014, respectively, for articles related to refurbishing by Samsung, Nintendo, and Apple). However, several aspects make the product line decision to include refurbished products a complex one. First, since the new and refurbished versions of a product are substitutes, refurbished units may cannibalize demand for new units (Atasu et al. 2010, Guide and Li 2010, Ovchinnikov 2011). Indeed, an increasing number of consumers consider buying refurbished versions of products rather than their more expensive new counterparts (evident from various discussion forums such as DPReview 2012, MacRumors 2012, Reddit 2016, etc.). In this quest, they are also aided by emerging shopping tools such as https://refurb-tracker.com and https://www.refurb.me (Broida 2014, Clover 2019). Second, the availability of refurbished units may lag new product launch because of the multiple steps involved, such as handling returns, diagnosing errors, refurbishing, and reselling (Guide et al. 2006). Since refurbished units are derived from units previously sold as new, the quantity of new units produced constrains the quantity of refurbished units that can be offered. Finally, consumers are becoming more sophisticated and strategic (i.e. forward-looking) in making their purchase decisions (Li et al. 2014, Su 2007). Strategic consumers make their purchase decisions based on not only what is offered today, but also what they expect would be offered in the future (Besanko and Winston 1990, Jerath et al. 2010). While the issues of cannibalization and the quantity linkage between new and refurbished products have been considered in prior research (Atasu et al. 2008, Ferguson and Toktay
2006, Ferrer and Swaminathan 2006, Orsdemir et al. 2014), the strategic behavior of consumers has not been investigated in the context of DPRs and their potential refurbishing. Our work addresses this gap by examining how DPRs and their potential refurbishing influence the intertemporal product strategy of a firm offering an innovative product to strategic consumers.

The issue of strategic consumers is pertinent to the context of firms offering refurbished versions of products because consumers are known to act strategically in anticipation of the availability of refurbished units in the future (evident from discussion forums such as those cited above). Strategic behavior of consumers exposes a firm to the “time inconsistency” problem: If consumers can foresee that the firm has an incentive to drop prices in the future, their willingness to pay for the product today will be lower, resulting in a reduction in the firm’s total profit (Bulow 1982, Coase 1972). More generally, the time inconsistency problem arises when a firm’s future decisions (including the price of a product in the future) affect the value of the product sold today, and the firm cannot credibly commit to future decisions (Waldman 2003). Paradoxically, the very flexibility of a firm in making decisions across time can hurt the firm. In such situations, more flexibility (in terms of the number of decisions or the feasible range of a decision variable) could be less desirable for the firm.

To the best of our knowledge, the impact of the refurbishing of DPRs on a firm’s time inconsistency problem has not been examined in prior research. On the one hand, DPRs may exacerbate the time inconsistency problem because these returns provide the firm with an incentive to offer the refurbished product in addition to or in place of the new product in the future (i.e., this additional flexibility afforded by the refurbishing of DPRs could worsen the time inconsistency problem). On the other hand, DPRs may mitigate the time inconsistency problem since the refurbished product can substitute demand away from the new product, thereby weakening the firm’s incentive to offer the new product in the future. Thus, it is unclear how DPRs and their potential refurbishing influence a firm’s intertemporal product strategy in the presence of strategic consumers. Accordingly, the following questions summarize the focus of our research:

1. What is the optimal intertemporal product strategy of a firm that faces strategic consumers and can refurbish DPRs?

2. How are the firm’s product line decisions influenced by the defect rate and the perceived quality of refurbished DPRs?

3. How does the defect rate affect the firm’s profit?

To answer these questions, we develop a two-period game-theoretic model where consumers are strategic and the firm cannot credibly commit to its second-period decisions in the first period. In the first period, the firm introduces a new product and decides its price. Consumers, who are utility-maximizing, strategic, and
heterogeneous in their valuations of product quality, make their purchase decisions taking into account not only the net utility from buying in the first period but also the anticipated (future) net utility from buying in the second period. In the second period, given the sales quantity and DPRs in the first period, the firm decides the prices of the new and the refurbished products. Consumers who remain in the market at the end of the first period make their purchase decisions after observing the second-period prices. In both periods, consumers return defective units to the firm, and the firm, in turn, provides these consumers with replacements (in extensions of our main model, we consider consumers opting for a refund instead of a replacement and the setting where uncertainty in the defect rate may result in the firm’s production quantity being insufficient to provide a replacement for every defective unit).

A novel contribution of our work is the recognition of DPRs as a possible commitment device. Our analysis shows that, as the defect rate increases beyond a threshold, the firm’s incentive to refurbish DPRs results in the reduction and, eventually, elimination of the firm’s incentive to offer the new product in the second period. Furthermore, we show that the negative impact of DPRs on the firm’s profit is alleviated because the refurbishing of DPRs helps the firm implicitly commit to limiting the quantity of the new product offered in the second period, thereby allowing the firm to charge a premium for the new product in the first period. As a result, the firm’s profit may even increase with the defect rate.

Prior literature on consumer returns policies shows that a moderate return rate may be optimal for the firm because allowing returns helps reduce consumers’ purchase risk arising from uncertainty about utility or fit (e.g., Ketzenberg and Zuidwijk 2009, Petersen and Kumar 2009, Su 2009). However, our analysis provides an entirely different explanation for the possible increase in firm profit with the product return rate. For managers who are wary of quality issues with innovative products, our analysis shows that the negative impact of DPRs may be overstated. While our results do not imply that some defects is better than no defects, they do indicate that opportunities to achieve marginal reductions in defect rates may not be worth the investment, and may even be counterproductive. Our analysis also suggests that efforts towards enhancing the perceived quality of the refurbished product or decreasing the hassle cost for consumers may better serve the firm than efforts towards marginally improving the conformance quality of the new product. Importantly, our qualitative findings persist across a range of extensions of our main model.

The rest of the paper is organized as follows. In §2, we position our research in the context of the relevant literature. We introduce our model in §3 and derive the firm’s equilibrium product strategy in §4. In §5, we discuss the role that DPRs may play in mitigating the firm’s time inconsistency problem, and the resulting non-monotonic behavior of the firm’s profit with respect to the defect rate. §6 presents the setting where the defect rate is uncertain when the product is introduced, resulting in either leftover inventory of the new product from the first period due to overproduction, or a stockout due to underproduction. We conclude with
a summary of our findings and a discussion of managerial insights in §7. Proofs of results in §3, §4, and §5 are included in the Online Appendix A. Appendices B through F are available in the unabridged version of this paper (Singh et al., 2020).

2. Literature Review

This paper considers the product strategy of a firm that faces strategic consumers and that can refurbish defective product returns (DPRs). Thus, our work is related to two streams of research: (1) closed-loop supply chains (CLSCs), wherein the refurbishing of product returns has received significant attention; and (2) the durable goods literature, which examines the challenges of selling durable products over multiple periods.

The internal competition between new and refurbished products (derived from either end-of-use returns or consumer returns) is an essential concern addressed in the CLSC literature (e.g., Guide and Li 2010, Vorasayan and Ryan 2006). The complexity in managing product returns lies in the fact that new and refurbished versions are not only substitutes but also complements of each other; the complementarity arises because the quantity of cores available for refurbishing is constrained by the quantity of new units sold previously (e.g., Atasu et al. 2008, Ferrer and Swaminathan 2006, Savaskan et al. 2004). In fact, if refurbishing is sufficiently attractive, the firm might deliberately underprice the new product in order to generate more returns (Debo et al. 2005). The CLSC literature has also studied how a firm’s strategy for new and refurbished products may depend on potential competition from third-party refurbishers (e.g., Ferguson and Toktay 2006, Majumder and Groenevelt 2001).

Within the CLSC literature, a stream of papers has considered the refurbishing of consumer returns. Unlike end-of-use returns, consumer returns are generated close to the time of sale due to reasons such as defects or lack of fit. Ketzenberg and Zuidwijk (2009) examine the optimal pricing and returns policy in a setting where the firm faces uncertain demand and where consumer returns can be restored to like-new condition and resold. Ferguson et al. (2006) investigate the problem of false failure returns and propose a target rebate contract that incentivizes the retailer to increase its effort, thus decreasing the amount of false failure returns. Both Ketzenberg and Zuidwijk (2009) and Ferguson et al. (2006) consider the refurbishing of consumer returns but assume an exogenously-given value of refurbished products. Pince et al. (2016) study how an OEM should allocate consumer returns between fulfilling warranty claims and remarketing refurbished products; the prices of the new and the refurbished products, as well as the quantity of returns to be used for warranty claims, are jointly optimized. Calmon et al. (2019) examine the problem of a social enterprise distributing a life-improving product through a retailer to risk-averse consumers, and model the information accuracy and refund policy offered to consumers. They find that improving information accuracy and investing in reverse logistics to improve the salvage value of consumer returns (e.g., through refurbishing) are strategic
substitutes in determining product adoption. While Calmon et al. (2019) endogenize the refund policy but assume an exogenous salvage value of returns, we assume an exogenous defect rate but endogenize the prices of the new and the refurbished products offered across time. In addition to endogenizing the prices of both the new product and refurbished DPRs in an intertemporal setting, we also consider consumers to be strategic – which, as discussed earlier – is pertinent to the context of firms offering new and refurbished versions of products across time.

The issue of strategic consumer behavior, wherein the purchase decisions of consumers are influenced by their anticipation of the firm’s product strategy in the future, has been of longstanding interest in the durable goods literature (Waldman 2003). While a durable goods manufacturer might announce that it will not continue production in the future, it has the incentive to produce additional units, reduce prices, and attract new consumers when that future arrives (Coase 1972). This inconsistent behavior of the firm over time, known as “time inconsistency,” negatively affects the profit of the firm if consumers are strategic (Bulow 1982, Stokey 1981). The consideration of strategic consumers is pertinent to CLSCs because consumers are increasingly aware and informed that returns arising from sales of the new product today may cause the firm to offer refurbished units in the future. However, the impact of strategic consumer behavior – and its implications for a firm’s intertemporal product strategy – has not received sufficient attention in the CLSC literature (exceptions include Oraiopoulos et al. 2012 and Zhang and Zhang 2018). Though Oraiopoulos et al. (2012) regard consumers to be strategic, they consider end-of-use returns and examine a different setting wherein the OEM (who does not participate in remanufacturing) charges a relicensing fee to a third-party remanufacturer. Zhang and Zhang (2018) study how different intensities of strategic customer behavior impact the economic and environmental values of trade-in remanufacturing in the presence of uncertain consumer valuations, and where new and remanufactured products (derived from end-of-use returns) are indistinguishable to consumers. In contrast, we examine the firm’s product line decision over time to include refurbished DPRs in the future, while allowing for a lower perceived quality of refurbished units (compared to new units).

The durable goods literature has proposed several commitment devices to mitigate the time inconsistency problem. These include leasing as opposed to selling (Bulow 1982, Desai and Purohit 1998), planned obsolescence (Bulow 1986, Waldman 1993), and the choice of product architecture (Ramachandran and Krishnan 2008). In our setting, DPRs provide the firm with an opportunity to offer the refurbished product in place of the new product in the future. Thus, a contribution of our work is the recognition of DPRs as a possible commitment device that can mitigate the firm’s time inconsistency problem.
3. Model

We consider a two-period model to characterize the temporal separation between the introduction of a new product and the availability of its refurbished version, and to capture the strategic (i.e., forward-looking) behavior of consumers. In the first period, the firm sells the new product. A fraction $\alpha' \in (0, 1)$ of new units are defective; we assume that this defect fraction is the same in each period (in Appendix D.1, we consider a lower defect fraction in the second period). The firm allows consumers to exchange a defective unit for another new unit. In the second period, the firm can refurbish defective product returns (DPRs) from the first period and sell the new and/or the refurbished product. The defect fraction of the refurbished product is $k'_r \alpha'$ such that $k'_r \in [0, 1)$, which is consistent with the observation that refurbished products typically go through individual quality checks whereas new products are typically tested by random sampling (Atasu et al. 2008, Consumer Reports 2018). We assume that the defect rates for the new and the refurbished products are also known to consumers from information in product-specific discussion forums (e.g., MacRumors 2012), annual reports (for publicly traded firms; typically 10-Ks), and industry reports (e.g., Business Wire 2011, Koenig 2014). In §6, we extend our analysis to the setting where the defect rate is uncertain in the first period. Without loss of generality, we assume that the firm can costlessly dispose of DPRs that are not refurbished (in Appendix D.2, we consider a positive salvage value of unrefurbished DPRs). We restrict our analysis to the parameter region where the discounted two-period profit of the firm is positive.

Products: We denote the perceived qualities of the new and the refurbished products by $v_n$ and $v_r$, respectively. Also, we assume $v_n > v_r$ since it has been empirically shown that consumers have a lower willingness to pay for refurbished products compared to new products (Guide and Li 2010, Subramanian and Subramanyam 2012). We denote the prices of the new product in the first period, the new product in the second period and the refurbished product (offered only in the second period) by $p_1$, $p_2$ and $p_r$, respectively, and the realized sales quantities (i.e., quantity produced minus DPRs) by $q_1$, $q_2$, and $q_r$, respectively. The marginal production cost for the new product is $c_n$ in each period. The marginal cost of refurbishing a DPR unit, denoted by $c_r$, is lower than the marginal cost of producing a new unit, i.e., $0 \leq c_r < c_n$ (Ferguson and Toktay 2006, Guide et al. 2006, Pince et al. 2016, Savaskan et al. 2004).

Consumers: Each consumer demands at most one of the products (new or refurbished). Consumers are heterogeneous in their valuations of product quality: A consumer of type $\theta$ receives utility $v\theta$ from a product of perceived quality $v$. We assume a continuum of consumers wherein the quality valuation $\theta$ is uniformly distributed between zero and one, and we normalize the total market size to one (in Appendix D.3, we discuss the setting where new consumers enter the market in the second period). When exchanging a defective unit for
another unit, a consumer incurs a hassle (or transaction) cost (Davis et al. 1995, McWilliams 2012, Moorthy and Srinivasan 1995). The hassle cost for a consumer may consist of one or more of the following: efforts and inconvenience in placing the defective unit back in the original packaging, retrieving purchase receipts, visiting the store again, and possibly having to convince the firm of no mishandling (Davis et al. 1995, Davis et al. 1998). Let \( h \) denote the hassle cost for a consumer in each instance of returning a defective unit. We restrict our analysis to the parameter regions where \( h \) is less than the equilibrium prices of the products, in order to preclude the situation where consumers would forego a replacement or a refund and not return a defective unit.

Net Utilities: The expected net utility of a consumer of type \( \theta \) from buying a new product of perceived quality \( v_n \) with defect fraction \( \alpha' \) at price \( p_n \) is:

\[
U_\theta^n = -p_n + v_n\theta (1 - \alpha') + \alpha' [v_n\theta (1 - \alpha') - h + \alpha' (v_n\theta (1 - \alpha') - h + ...) ,
\]

which yields \( U_\theta^n = v_n\theta - h\alpha' / (1 - \alpha') - p_n \). For exposition, we define \( \alpha := \alpha' / (1 - \alpha') \) and henceforth refer to \( \alpha \) as the defect rate for the new product (unlike \( \alpha' \), the defect rate \( \alpha \) can be greater than 1).

Thus, \( U_\theta^n = v_n\theta - h\alpha - p_n \). Similarly, the expected net utility for a consumer of type \( \theta \) from buying a refurbished product of perceived quality \( v_r \) with defect rate \( k_r\alpha \) at price \( p_r \) is: \( U_\theta^r = v_r\theta - h k_r\alpha - p_r \), where \( k_r\alpha = k_r'\alpha' / (1 - k_r'\alpha') \). Note that \( k_r' \in [0, 1) \implies k_r \in [0, 1) \). Since a consumer of type \( \theta \) would have purchased a new [refurbished] unit only if \( U_\theta^n > 0 \) [\( U_\theta^r > 0 \)], the consumer would obtain a higher expected net utility from replacing the unit if it were defective, over returning it for a refund (or doing nothing). Nonetheless, in Appendix D.4, we consider the setting where consumers may opt for a refund instead of a replacement.

Since consumers are strategic, they make their purchase decisions with the objective of maximizing their expected discounted net utility. The firm and consumers discount second-period profits and utilities, respectively. Whereas, in our main analysis, we consider the firm and consumers to have the same discount factor \( \rho \in (0, 1) \), in Appendix E, we characterize and discuss the equilibrium solutions for the generalized model where the firm and the consumers may have different discount factors (including the analysis of corner cases where the discount factors may equal 0 or 1). We denote the expected discounted net utilities for a consumer of type \( \theta \) from buying: (1) the new product in the first period; (2) the new product in the second period; (r) the refurbished product in the second period; or, (0) none of the products, by \( U_\theta^1, U_\theta^2, U_\theta^r, \) and \( U_\theta^0 \), respectively. Without loss of generality, we set \( U_\theta^0 \) to zero. Thus, the expected discounted net utilities for a consumer of type \( \theta \), depending upon which product is purchased and when, are:

\[
U_\theta^1 = v_n\theta - h\alpha - p_1; \quad U_\theta^2 = \rho (v_n\theta - h\alpha - p_2); \quad U_\theta^r = \rho (v_r\theta - h k_r\alpha - p_r); \quad U_\theta^0 = 0. \quad (3.1)
\]
Since we assume a continuum of consumers, in order to realize a sales quantity $q$ of the new [refurbished] product, the firm should produce a quantity $(1 + \alpha) q$ [refurbish a quantity $(1 + k_r \alpha) q$ of DPRs] (see Lemma 1 in Appendix A.1, which allows us to drop the use of the prefix “expected” when we refer to costs, revenues, sales quantities, and firm profit in §4 and §5). Thus, the effective marginal costs for the new and the refurbished products are $(1 + \alpha) c_n$ and $(1 + k_r \alpha) c_r$, respectively. Also, we assume that the hassle cost $h$ is not so large that a consumer’s net utility from the refurbished product would exceed that from the new product if both products were offered at the same price in the second period. Specifically, we assume that $h < \bar{h} := \left( v_n - v_r \right) c_n (1 + \alpha) \frac{\alpha}{v_n - v_r k_r}$ if $k_r < \frac{v_n}{v_r}$. No such bound on $h$ is needed if $k_r \geq \frac{v_n}{v_r}$.

4. Analysis

4.1 Dynamics of the Game

**Sequence of Decisions:** At the beginning of the first period, the firm sets the price of the new product $(p_1)$. Subsequently, consumers decide whether to buy the new product in the first period or wait until the second period, based on the anticipated availability and prices of the new and the refurbished products in the second period. In the second period, the firm decides which products to offer (new, refurbished, or both), and at what prices. To obtain a subgame perfect Nash equilibrium, we solve the dynamic game between the firm and consumers by backward induction, starting with the second period.

**Second-Period Decisions**

In the second period, consumers who do not buy in the first period make their purchase decisions after observing prices $p_2$ and $p_r$ for the new and the refurbished products, respectively. Note that $q_1$ consumers have exited the market after purchasing the new product in the first period. In the second period, a consumer of type $\theta$ (who did not buy in the first period) buys the new product if $U_2^\theta \geq \max \{U_2^\theta, 0\}$ and buys the refurbished product if $U_r^\theta \geq \max \{U_2^\theta, 0\}$. Given the sales quantity $q_1$ in the first period and prices $p_2$ and $p_r$, the resulting sales quantities of the new and the refurbished products in the second period are:

$$
q_2 (q_1, \mathbb{P}) = 1 - q_1 - \max \left\{ \frac{p_2 + ha - (p_r + h k_r \alpha)}{v_n - v_r}, \frac{p_2 + h \alpha}{v_n} \right\},
$$

$$
q_r (q_1, \mathbb{P}) = \min \left\{ 1 - q_1, \frac{p_2 + ha - (p_r + h k_r \alpha)}{v_n - v_r} - \frac{p_r + h k_r \alpha}{v_r} \right\}.
$$

(4.2)

where $\mathbb{P}$ is the vector of prices $\{p_2, p_r\}$.

Let $\Pi_2 (q_1, \mathbb{P})$ denote the second-period profit of the firm for a given first-period sales quantity $q_1$ of the new product and the price vector $\mathbb{P}$. The firm’s optimization problem in the second period is given by:
\[
\begin{align*}
\max_{(p_1, p_2)} \Pi_2 (q_1, P) &= [q_2 (q_1, P) (p_2 - (1 + \alpha)c_n) + q_r (q_1, P) (p_r - (1 + k_r\alpha)c_r)] \\
\text{s.t. } q_2 (q_1, P) &\geq 0, \ \alpha q_1 \geq (1 + k_r\alpha) q_r (q_1, P) \geq 0.
\end{align*}
\] (4.3)

The constraint \((1 + k_r\alpha) q_r \leq \alpha q_1\) arises in (4.3) since refurbished units are derived from DPRs generated by new product sales in the first period. We use asterisks to denote optimal/equilibrium solutions. We denote the optimal second-period profit and prices for the new and the refurbished products by \(\Pi_2^* (q_1), p_2^* (q_1),\) and \(p_r^* (q_1),\) respectively, and the resulting net utilities for a consumer of type \(\theta\) from buying these products by \(U_2^{\theta*}\) and \(U_r^{\theta*},\) respectively.

**First-Period Decisions**

In the first period, consumers make their purchase decisions after observing price \(p_1\) and anticipating the availability and prices of the new and the refurbished products offered in the second period. In a subgame perfect Nash equilibrium, consumers’ anticipated second-period prices for the new and the refurbished products are equal to \(p_2^* (q_1)\) and \(p_r^* (q_1),\) respectively. Thus, a consumer buys the new product in the first period if \(U_1^\theta \geq \max \{U_2^{\theta*}, U_r^{\theta*}, 0\},\) yielding the following relationship:

\[
q_1 (p_1) = 1 - \max \left\{ \frac{2 (p_1 + h\alpha) - \rho (c_n (1 + \alpha) + h\alpha)}{v_n (2 - \rho)}, \frac{p_1 + h\alpha}{v_n}, \right. \\
\left. \min \left\{ \frac{2 (p_1 + h\alpha) - \rho (c_r (1 + k_r\alpha) + h k_r\alpha)}{2v_n - \rho v_r}, \frac{1 - (1 + k_r\alpha) (v_n - h\alpha - p_1)}{v_n (1 + k_r\alpha) + \rho h k_r\alpha} \right\} \right\}
\] (4.4)

The firm’s objective in the first period is to maximize its total profit over the two periods by setting price \(p_1\) for the new product, taking into account the resulting sales quantity \((q_1)\) and the optimal second-period prices \(p_2^* (q_1)\) and \(p_r^* (q_1).\) Let \(\Pi (p_1)\) denote the total discounted profit of the firm from selling the new product in the first period, and the new product and/or the refurbished product in the second period. Thus, the firm’s problem at the beginning of the first period is:

\[
\max_{p_1} \Pi (p_1) = [q_1 (p_1) (p_1 - (1 + \alpha)c_n) + \rho \Pi_2^* (q_1) (p_1))] \\
\text{s.t. } q_1 (p_1) \geq 0.
\] (4.5)

Table 1 summarizes the possible product strategies of the firm over the two periods. We denote these strategies by the triad \(WXY,\) where \(W \in \{N, \emptyset\}, X \in \{N, \emptyset\},\) and \(Y \in \{R_S, R_A, \emptyset\}\) denote whether or not the firm offers the new product in the first period, the new product in the second period, and the refurbished product in the second period, respectively. The subscript for \(R\) denotes whether some \((S)\) or all \((A)\) of the first-period DPRs are refurbished. For exposition, we denote \(\mu_n := v_n - (1 + \alpha)c_n - h\alpha\) and \(\mu_r := v_r - (1 + k_r\alpha)c_r - h k_r\alpha.\) Note that, in the first period, the firm has an incentive to offer the new product.
only if \( \mu_n + \rho \left( \frac{\alpha}{1 + k r \alpha} \right) \mu_r > 0 \). In the second period, the firm has an incentive to offer the new product only if \( \mu_n > 0 \), and the refurbished product only if \( \mu_r > 0 \).

We characterize the firm’s and consumers’ equilibrium decisions over the two periods in two steps: In §4.2, we derive the firm’s optimal product strategy in the second period as a function of the sales quantity in the first period. We then derive the equilibrium decisions in the first period in §4.3. This approach ensures that we obtain a subgame perfect equilibrium where consumers, who are strategic, foresee the firm’s dynamic incentives. Note that, depending on the quantity of DPRs available for refurbishing and the profitability of offering the refurbished product, the firm is able to optimally reduce [increase] the second period demand for the refurbished product by increasing [decreasing] \( p_r \). In turn, since consumers are strategic, a consumer in the first period anticipates: (i) the quantity of returns available for refurbishing in the second period; (ii) the firm’s second-period optimal prices; and, (iii) the resulting net utilities, depending upon which product is purchased. Effectively, in equilibrium, there is no stockout in the model that we analyze here. However, stockouts are a possibility in the model that we consider in §6, wherein the defect rate is uncertain and the production quantity needed to meet demand is not known with certainty.

### 4.2 Second Period Optimization

The state of the market at the start of the second period is characterized by the sales quantity of the new product in the first period \( q_1 \). The firm may choose one of the following four product strategies in the second period: offer both the new and the refurbished products (Product Line); offer only the refurbished product (Refurbished Only); offer only the new product (New Only); and offer none of the products (None). Proposition 1 characterizes the conditions under which different product strategies are optimal in the second period, depending on the first-period sales quantity \( q_1 \) and the defect rate \( \alpha \).

**Proposition 1. Second-Period Product Strategy.**

There exist \( \bar{\alpha}, \bar{q}_1 \), and \( \bar{q}_1 \) (with \( \bar{q}_1 < \bar{q}_1 \)), such that in the second period it is optimal to offer:

(i) none of the products if \( q_1 \geq \bar{q}_1 \) (None);

(ii) only the new product if \( \alpha \leq \bar{\alpha} \) and \( q_1 < \bar{q}_1 \) (New Only);

(iii) only the refurbished product if \( \alpha > \bar{\alpha} \) and \( q_1 \leq q_1 < \bar{q}_1 \) (Refurbished Only); and,

(iv) both the new and the refurbished products if \( \alpha > \bar{\alpha} \) and \( q_1 < \bar{q}_1 \) (Product Line).

The thresholds \( \bar{\alpha}, \bar{q}_1, \) and \( \bar{q}_1 \) are defined in the proof of Proposition 1.

**Proof.** See Appendix A.2.

The optimal prices and sales quantities in the second period (as functions of \( q_1 \)) for the different product strategies are provided in Table 2, and the conditions under which these product strategies are optimal are
provided in Table 3. As illustrated in Figure 1, if the firm has saturated the market by selling a substantial quantity of the new product in the first period (i.e., $q_1 \geq \bar{q}_1$), then the remaining consumers’ willingness to pay in the second period is sufficiently low that it is not profitable for the firm to sell either the new or the refurbished product, even if a large quantity of DPRs may be available for refurbishing (i.e., Proposition 1(i)). In other words, the firm offers one or both of the products (new/refurbished) in the second period only if the quantity of the new product sold in the first period is below a threshold (i.e., $q_1 < \bar{q}_1$).

![Figure 1: Second-Period Strategy as a Function of First-Period Sales Quantity $q_1$ and Defect Rate $\alpha$](image)

If $q_1 < \bar{q}_1$ and the defect rate is lower than the threshold $\bar{\alpha}$, the firm – despite the availability of DPRs for refurbishing – offers only the new product in the second period (i.e., Proposition 1(ii)). This is because when $\alpha \leq \bar{\alpha}$, the effective marginal cost ($c_n (1 + \alpha)$) and the hassle cost of returns ($h \alpha$) for the new product are both sufficiently low. This implies not only a sufficiently low cost to the firm for offering the new product but also a sufficiently high consumer willingness to pay. And, if the refurbished product were also offered, the net revenue from the refurbished product would be insufficient to offset the loss from cannibalization of new product demand.

However, if the defect rate is higher than the threshold $\bar{\alpha}$, then, in the second period, it becomes profitable for the firm to offer the refurbished product. This is because of the higher effective marginal cost and higher hassle cost of returns for the new product compared to the refurbished product (i.e., $c_n (1 + \alpha)$ and $h \alpha$, compared to $c_r (1 + k_r \alpha)$ and $h k_r \alpha$, respectively, where $c_r < c_n$ and $k_r < 1$). The sales quantity of the new product in the first period determines whether, in the second period, the firm should offer the new product in addition to the refurbished product: If $q_1 \geq \frac{q_1}{2}$, the remaining consumers’ willingness to pay is relatively low and the supply of DPRs from the first period is abundant. As a result, the firm offers only the refurbished
product (i.e., Proposition 1(iii)). However, if $q_1 < q_{1*}$, the firm offers the new product as well because the market is not as saturated from sales in the first period, and the willingness to pay of the remaining consumers is reasonably high (i.e., Proposition 1(iv)).

4.3 Complete Two-Period Solution

In the previous section, we characterized the firm’s optimal second-period product strategy for a given first-period sales quantity ($q_1$). In this section, we derive the equilibrium product strategy for the firm over both periods by solving the complete two-period game between the firm and consumers. Proposition 2 presents the firm’s equilibrium product strategy over both periods. Table 4 summarizes the equilibrium prices and sales quantities of the new product in the first period for the product strategies that are plausible in equilibrium, and Table 5 summarizes the conditions under which these product strategies emerge in equilibrium.

Proposition 2. Equilibrium Product Strategy over Both Periods.

The firm always offers the new product in the first period.

Further, there exist $v^N_r$ and $v^R_r$ (with $v^N_r < v^R_r$) such that the firm’s strategy in the second period is to offer:

(i) only the new product if $v_r \leq v^N_r$ (New Only);

(ii) both the new and the refurbished products if $v^N_r < v_r < v^R_r$ (Product Line); and,

(iii) only the refurbished product if $v_r \geq v^R_r$ (Refurbished Only).

The thresholds $v^N_r$ and $v^R_r$ are defined in the proof of Proposition 2.

Proof. See Appendix A.3.

Due to the discounting of future profit, postponing the introduction of the product to the second period is suboptimal for the firm. Therefore, the firm always offers the new product in the first period. Furthermore, inducing all prospective consumers to buy in the first period itself would require the firm to set a suboptimally low price. Therefore, in equilibrium, the firm offers at least one (new, or refurbished, or both) of the products in the second period (formally shown in Lemma 2 in Appendix A.3). Figure 2 depicts the firm’s equilibrium product strategy in the second period (derived in Proposition 2).

When the perceived quality of the refurbished product ($v_r$) and the defect rate ($\alpha$) are both low, offering the new product in the second period is more profitable than offering the refurbished product. This is due to not only the relatively low consumer willingness to pay for the refurbished product, but also a low effective marginal cost ($c_n (1 + \alpha)$) and hassle cost of returns ($h\alpha$) for the new product. Thus, when $v_r$ and $\alpha$ are both low, the firm offers only the new product in the second period.

In contrast, when both $v_r$ and $\alpha$ are sufficiently high, not only do consumers perceive the refurbished product to be closer in quality to the new product, but also the effective marginal cost and the hassle cost
of returns for the new product are sufficiently higher in comparison to the refurbished product. Therefore, in the second period, the firm offers only the refurbished product in this region.

In all other situations, the firm offers the product line in the second period, comprising both the new and the refurbished products. When both $v_r$ and $\alpha$ are in an intermediate range, or when $v_r$ is low but $\alpha$ is high, neither product dominates the other in terms of either net utility to consumers or effective marginal costs, and the firm offers both products. The product line is also optimal if $v_r$ is high and $\alpha$ is low: Here, the sales quantity of the refurbished product is limited by the low volume of DPRs generated in the first period. Therefore, although $v_r$ is high, the new product is included as part of the product line in the second period to capitalize on unmet consumer demand.

To summarize, in equilibrium, the firm always offers the new product in the first period and at least one version of the product — new or refurbished — in the second period. In particular, for a sufficiently high defect rate and perceived quality of the refurbished product, the firm offers only the refurbished product in the second period.

5. The Cost (or Value?) of Defective Product Returns

In this section, we examine the overall effect of DPRs on firm profit and describe the underlying mechanism that may make DPRs valuable.
5.1 Effect of Defect Rate on Firm Profit

It is reasonable to expect that the firm’s profit would monotonically decrease as the defect rate increases. This is because a higher defect rate leads to direct costs for the firm in production and refurbishing: (a) the firm failed to earn revenue from a DPR unit when it was initially sold; (b) if the firm does not refurbish a DPR unit from the first period, it earns nothing; if the firm does refurbish a DPR unit from the first period, it incurs an additional effective marginal cost of \( c_r (1 + k_r \alpha) \) to refurbish the unit, in addition to the refurbished unit having a lower perceived quality compared to a new unit; and, (c) the hassle cost of returning a defective unit lowers a consumer’s willingness to pay for the product (new or refurbished) in the first place. However, as we show in Proposition 3 below, the firm’s profit may be non-monotonic in the defect rate.

**Proposition 3. Defect Rate, Time Inconsistency Problem, and Firm Profit.**

(i) If the firm could credibly commit to its future decisions, its profit \( \Pi^C \) would monotonically decrease in the defect rate \( \alpha \).

(ii) However, in our setting, where the firm cannot credibly commit to its future decisions, firm profit \( \Pi^* \) is bounded above by \( \Pi^C \), and,

(a) there exists a threshold defect rate, \( \alpha_c \), beyond which firm profit in our setting equals that in the commitment scenario (i.e., \( \Pi^* = \Pi^C \));

(b) there exists \( \hat{\nu}_r \) such that firm profit in our setting \( \Pi^* \) is non-monotonic in the defect rate \( \alpha \) when \( \hat{\nu}_r < \nu_r < \nu_n \).

**Proof.** See Appendix A.4.

Proposition 3(ii)(b) states that an increase in the defect rate does not necessarily reduce the firm’s profit when it cannot credibly commit to its future decisions. Why do DPRs have this counterintuitive effect? To understand this effect, we examine what the firm would do if it were able to commit to its second-period decisions (we discuss this “commitment scenario” in detail in Appendix B): In essence, the firm would commit to offer the new product only in the first period and possibly offer the refurbished product in the second period (Proposition 7 in Appendix B). In this setting, the firm’s profit would monotonically decrease with the defect rate (Proposition 3(i)) because DPRs – as discussed above – add costly friction to both production and demand.

In reality, though, such commitments are rarely credible because of the firm’s inherent incentive to offer the new product again in the second period, resulting in a lower firm profit than in the commitment scenario (Coase 1972). However, in our setting, as the defect rate increases, the firm’s incentive to offer the new product in the second period diminishes. Beyond a threshold defect rate \( \alpha_s \) in Figure 3, where the firm’s equilibrium strategy changes from \( NNRA \) (New Product in Period 1 – New Product in Period 2 – Refurbish All DPRs) to
(New Product in Period 1 – No New Product in Period 2 – Refurbish All DPRs)), the firm offers the new product exclusively in the first period and is able to charge higher prices for both the new product in the first period and the refurbished product in the second period. This effectively allows the firm to overcome its inability to credibly commit to its future decisions. Thus, when the defect rate is sufficiently high, the firm’s profit in our setting equals that in the commitment scenario (Proposition 3(ii)(a)).

Furthermore, when the perceived quality of the refurbished product is sufficiently high, the firm’s profit is non-monotonic in the defect rate (Proposition 3(ii)(b)). Specifically, the firm’s profit increases with the defect rate within a certain interval, denoted by \((\alpha_1, \alpha_2)\). For example, in the parameter setting considered in Figure 3, firm profit increases with the defect rate in the interval \(\sim (12.7\%, 16.6\%)\). In this interval, the additional DPRs are so effective in displacing the new product in the second period that the firm’s profit is boosted by the premium it can charge for the new product in the first period. At the same time, the refurbished units can be sold at a higher margin due to the near elimination of the new product in the second period. This combination of positive factors outweighs the cost increases in the interval \((\alpha_1, \alpha_2)\). Appendix C presents our findings from a broad numerical study, using 5,000 parameter combinations sampled randomly from ranges based on empirical evidence. Notably, we find that in most of the parameter combinations, the firm’s profit could be non-monotonic in the defect rate.

Managerial Implications

Figure 4 depicts how the width of the interval \((\alpha_1, \alpha_2)\) changes with: (i) the perceived quality of the refurbished product \(v_r\); and, (ii) consumers’ hassle cost of returns \(h\). First, we observe that the interval is wider
when the perceived quality of the refurbished product is higher: For example, in Figure 4(a), the interval is \( \sim (15.3\%, 16.7\%) \) when \( v_r = 0.8 \) and expands to \( \sim (12.7\%, 16.6\%) \) when \( v_r = 0.9 \). Second, we observe that the interval gets wider as the hassle cost of returns decreases: For example, in Figure 4(b), the interval is \( \sim (13.0\%, 15.6\%) \) when \( h = 0.1 \) and expands to \( \sim (12.3\%, 17.6\%) \) when \( h = 0.0 \). Additionally, Proposition 4 characterizes the effects of \( v_r \) and \( h \) on the firm’s profit in the region where the profit increases in the defect rate \( \alpha \).

![Figure 4: Parameter Ranges in which Firm Profit Increases with the Defect Rate: (Effects of the perceived quality of the refurbished product (\( v_r \)) and the hassle cost (\( h \)); \( c_n = 0.7, \ c_r = 0.1, \ v_n = 1.0, \ \rho = 0.9 \)](a) \( k_r = 0.25, \ h = 0.05 \) (b) \( v_r = 0.9, \ k_r = 0.25 \)

**Proposition 4. Effects of \( v_r \) and \( h \) when Firm Profit Increases in the Defect Rate \( \alpha \).**

In the region where the firm’s profit increases in the defect rate \( \alpha \):

(i) the firm’s profit increases in the perceived quality \( v_r \) of the refurbished product; and,

(ii) the firm’s profit decreases in the hassle cost \( h \).

**Proof.** See Appendix A.5.

The managerial implications of Proposition 4 are that, efforts towards increasing the perceived quality of the refurbished product (increasing \( v_r \)) or decreasing the hassle cost for consumers (decreasing \( h \)) may be more profitable than marginal improvements to the conformance quality of the new product. In practice, consumer perceptions of the refurbished product can be influenced by communicating the meticulousness of refurbishing processes and sharing positive consumer reviews of refurbished units. Hassle costs for consumers could be reduced by helping them transfer user settings to the replacement units and making the return experience more empathetic and pleasant. Note that we do not imply that some defects is better than no defects. Our results indicate that opportunities to achieve marginal reductions in defect rates may not be worth the investment, and may even be counterproductive. Eliminating defects altogether may be very expensive or even infeasible in practice, especially in fast-paced industries such as consumer electronics, where a particular
product may be produced and sold during a relatively short period of time for the firm to be able to completely 
eliminate defects.

5.2 DPRs as a Commitment Device

As discussed earlier, the firm would ideally like to communicate the following product strategy to consumers: 
The new product will be offered exclusively in the first period and only the refurbished product may be offered 
in the second period (Proposition 7 in Appendix B). However, strategic consumers can foresee that the firm 
may have an incentive to deviate from this commitment in the second period; in other words, the firm’s 
incentives suffer from time inconsistency (Coase 1972). Broadly speaking, the time inconsistency problem 
arises when a firm’s future decisions (such as the price of a product) affect the value of the product sold today, 
and the firm cannot credibly commit to its future decisions (Waldman 2003). Thus, a greater flexibility for the 
firm in terms of the number of decision variables or the feasible range of a decision variable may, paradoxically, 
turn out to be undesirable for the firm when decisions are made across time. For our setting, we quantify the 
firm’s relative profit reduction ($\tau$) due to the time inconsistency problem as:

$$\tau = \frac{\Pi_C^{\text{C}} - \Pi^*}{\Pi_C^{\text{C}}},$$

where $\Pi_C^{\text{C}}$ denotes the firm’s profit if it were able to commit to its second-period decisions (discussed in detail 
in Appendix B).

If the option of refurbishing were absent, the firm would have one decision to make in the second period: 
the price of the new product. In such a situation, our results would be in line with Bulow (1982), and 
$$\tau = \frac{\Pi_C^{\text{C}} - \Pi^*}{\Pi_C^{\text{C}}} = \frac{\rho(1-\rho)}{4-3\rho}$$ 
would be invariant with respect to the defect rate. However, with the option of refurbishing DPRs, the firm needs to make two decisions in the second period: the price of the new product 
and the price of the refurbished product. Intuitively, this additional flexibility (i.e., the option to offer the 
refurbished product as well) for the firm in the second period should worsen the time inconsistency problem. 
Moreover, since the availability of DPRs for refurbishing may increase with the defect rate, it is reasonable to 
expect the time inconsistency problem to worsen as the defect rate increases.

Instead, we observe that the effect of the defect rate ($\alpha$) on the firm’s time inconsistency problem is more 
nuanced. As illustrated in Figure 5, the option to refurbish DPRs (curve with solid line) does worsen the time 
inconsistency problem as the defect rate initially increases. However, as the defect rate increases beyond the 
threshold $\alpha_{w}$, the time inconsistency problem starts getting mitigated because the firm’s incentive to offer the 
new product in the second period is sufficiently diminished. Eventually, beyond the threshold $\alpha_{c}$ (Proposition 
3(ii)(a)), the firm no longer has an incentive to offer the new product in the second period, resulting in the
elimination of the time inconsistency problem altogether (i.e., $\tau = 0$ for $\alpha \geq \alpha_c$). In contrast, if the option of refurbishing the DPRs were absent (dotted line), the magnitude of the time inconsistency problem would be invariant with respect to the defect rate. Thus, a key contribution of our work is identifying how the refurbishing of DPRs can mitigate the time inconsistency problem.

In Appendix D, we discuss the robustness of our findings to the following relaxations/extensions: a lower defect rate for the new product in the second period (§D.1); a positive salvage value of unrefurbished DPRs (§D.2); new consumers entering the market in the second period (§D.3); and, consumers opting for a refund instead of a replacement (§D.4). These findings include: (a) results on the firm’s equilibrium product strategy (analytical); (b) DPRs playing a role as a commitment device (analytical); and (c) possible non-monotonicity of firm profit with respect to the defect rate (numerical).

6. Stochastic Model: Uncertainty in the Defect Rate

In this section, we consider the defect rate to be uncertain when the firm sets the production quantity and price of the new product in the first period and consumers evaluate whether to purchase in the first period or postpone their purchase decision to the second period. In the second period, the firm and consumers know the defect rate with certainty. In our model setting, the incorporation of uncertainty in the defect rate transforms the problem to a price-setting newsvendor problem with two additional complexities: First, the salvage value of any leftover inventory of the new product and DPRs from the first period is endogenously determined by
the firm’s production and pricing decisions over the two periods. Second, the firm’s decisions must take into account the strategic behavior of consumers.

We capture the magnitude of uncertainty in the defect rate by the parameters $\Delta$ and $\sigma$. Specifically, the defect rate is $\alpha_L := \alpha - (1 - \sigma) \Delta$ (low) with probability $\sigma$, and $\alpha_H := \alpha + \sigma \Delta$ (high) with probability $1 - \sigma$, where $(1 - \sigma) \Delta < \alpha$. Thus, the expected defect rate in the stochastic model is $\alpha_e := \sigma \alpha_L + (1 - \sigma) \alpha_H = \alpha$, facilitating comparisons with the deterministic model considered in the preceding sections (wherein the defect rate is known with certainty). In the first period, the firm decides the production quantity (denoted by $Q_1$) and price ($p_1$) of the new product. Let $\theta_m$ denote the quality valuation of the marginal consumer who is indifferent between participating in the purchase in the first period and postponing the purchase to the second period. We use the term “participate” because the firm stocks out if its production quantity is insufficient to provide a replacement unit for every defective new unit that is purchased.

The quantity of consumers who initially purchase a new unit is $q_m := 1 - \theta_m$. If the firm stocks out and is unable to provide a replacement unit for every defective unit, we assume proportional rationing by the firm in providing replacement units (Su 2007, Ovchinnikov 2011), with all consumers participating in the purchase being equally likely to get replacements for defective units. Those consumers who do not get replacements receive a full refund of the purchase price, and re-enter the market in the second period. We denote this unmet demand in the first period (which may or may not be met in the second period) due to the firm stocking out, by $\chi_1 \geq 0$. On the other hand, if the firm’s production quantity is sufficient to provide replacements for all defective units, the firm ends up with leftover inventory (denoted by $I_1 \geq 0$) that is carried over to the second period.

Table 14 in Appendix F.4 summarizes the possible product strategies of the firm over the two periods. As before, to obtain a subgame perfect Nash equilibrium, we solve the game between the firm and consumers by backward induction, starting with the second period. We discuss the equilibrium second-period and first-period decisions in §6.1 and §6.2, respectively. §6.3 examines the effect of uncertainty in the defect rate on the firm’s time inconsistency problem, including whether the firm’s profit could increase in the (expected) defect rate even when the defect rate is uncertain. Overall, our analysis in this section leads to the following two key insights when the defect rate is uncertain:

1. **DPRs continue to serve as a possible commitment device**: Even when the defect rate is uncertain, we find that the option to refurbish DPRs may serve as a credible commitment mechanism that: (a) mitigates the firm’s time inconsistency problem; and, (b) results in the firm’s expected profit being possibly non-monotonic with respect to the defect rate.

2. **Uncertainty suppresses the value of DPRs**: However, when the defect rate is uncertain, the firm’s ability to use DPRs as a commitment device is tempered. This is because of the possibility of either leftover
inventory of the new product from the first period due to overproduction, or unmet demand due to underproduction. Both these possibilities induce the firm to offer the new product in the second period.

6.1 Second Period Decisions

In the second period, the defect rate \( \alpha \) is known with certainty (i.e., either \( \alpha = \alpha_L \) or \( \alpha = \alpha_H \)). Given proportional rationing in the event of a stockout in the first period, the state at the beginning of the second period can be characterized by the realization of the defect rate \( \alpha \), the first-period sales quantity \( q_1 \) of the new product, and the leftover inventory \( I_1 \) of the new product from the first period; i.e., \( S = \{ \alpha, q_1, I_1 \} \). In the second period, a consumer’s net utilities from buying the new and the refurbished products are similar to the corresponding utilities given in equation (3.1): A consumer of type \( \theta \) who did not buy in the first period buys the new product if \( U^{\theta}_{2} \geq \max \{ U^{\theta}_{r}, 0 \} \) and buys the refurbished product if \( U^{\theta}_{r} \geq \max \{ U^{\theta}_{2}, 0 \} \).

If \( \max \{ U^{\theta}_{2}, U^{\theta}_{r} \} < 0 \), then it is optimal for this consumer to not purchase in the second period. Note that in the second period, consumers can compute their net utilities after knowing the realization of the defect rate \( \alpha \) and the prices of the new and the refurbished products (if offered by the firm). However, the second period utility is stochastic from the vantage point of the first period, wherein the defect rate is uncertain when consumers decide whether to purchase in the first period or wait for the second period. We discuss this first period decision for consumers in §6.2.

The sales quantities of the new and the refurbished products in the second period can also be written in a form analogous to equation (4.2), noting that the sales quantity \( q_1 \) itself depends on the realized defect rate, and the price \( (p_1) \) and production quantity \( (Q_1) \) in the first period. The firm’s optimization problem in the second period is:

\[
\max_{\{p_2, p_r\}, \{\alpha, q_1, I_1\}} \Pi_2 (S, P) = q_2 (S, P) p_2 - \max \{ q_2 (S, P) (1 + \alpha) - I_1, 0 \} c_n + q_r (S, P) (p_r - c_r (1 + k_r \alpha)) \\
\text{s.t.} \quad q_2 (S, P) \geq 0, \quad \alpha q_1 \geq (1 + k_r \alpha) q_r (S, P) \geq 0,
\]

where \( P \) is the vector of prices \( \{p_2, p_r\} \), and \( q_2 (S, P) \) and \( \max \{ q_2 (S, P) (1 + \alpha) - I_1, 0 \} \) represent the sales quantity and production quantity, respectively, of the new product in the second period. Using asterisks to denote equilibrium solutions, the profit obtained by solving problem (6.6) is denoted by \( \Pi^*_2 (\alpha, q_1, I_1) \). At the beginning of the second period, we either have \( I_1 = 0 \) (if there is a stockout or perfectly balanced production and demand in the first period) or \( I_1 > 0 \) (if there is overproduction and, thus, leftover inventory at the end of the first period). When \( I_1 = 0 \), the firm’s optimal product strategies in the second period are as presented in Proposition 1 in §4.2. Proposition 5 characterizes the firm’s optimal product strategy in the second period when \( I_1 > 0 \) (the optimal second-period sales quantities and prices for the different product strategies when \( I_1 > 0 \) are provided in Table 15 in Appendix F.4).
Proposition 5. Second-Period Product Strategy when $I_1 > 0$ (Stochastic Model).

When $I_1 > 0$, there exist $q_1^*$ and $\bar{q}_1$ (with $q_1^* < \bar{q}_1$) such that, in the second period, the firm optimally offers:

(i) **No product.** None of the products are offered if $q_1 \geq \bar{q}_1$;

(ii) **New Only.** Only the new product is offered

(a) with $0 < q_2^* < \frac{I_1}{1+\alpha}$ if $\bar{q}_1 - \frac{2I_1}{1+\alpha} < q_1 < q_1^*$;

(b) with $q_2^* = \frac{I_1}{1+\alpha}$ if $\bar{q}_1 - \frac{2I_1}{1+\alpha} \leq q_1 \leq q_1^* - \frac{2I_1}{1+\alpha}$;

(c) with $q_2^* > \frac{I_1}{1+\alpha}$ if $\bar{q}_1 - \frac{2I_1}{1+\alpha} \leq q_1 \leq q_1^* - \frac{2I_1}{1+\alpha}$;

(iii) **Product Line.** Both the new and the refurbished products are offered

(a) with $q_3^* = \frac{I_1}{1+\alpha}$ if $q_1 > \bar{q}_1$ and $q_1^* \leq q_1 < \bar{q}_1 - \frac{2I_1}{1+\alpha}$;

(b) with $q_3^* > \frac{I_1}{1+\alpha}$ if $q_1 > \bar{q}_1$ and $q_1^* < q_1$.

The threshold $\bar{\alpha}$ is as defined in the proof of Proposition 1.

**Proof.** See Appendix F.1.

Proposition 5 extends the results from Proposition 1 to the setting where $I_1 > 0$. Note that, in the deterministic model considered in the preceding sections (wherein the defect rate is known with certainty), it is neither profitable for the firm to have unmet demand in the first period nor is it profitable for the firm to produce in excess of demand and carry over inventory to the second period.

Similar to Proposition 1, we find that, under uncertainty in the defect rate, it is not profitable to offer either the new or the refurbished product in the second period if the market was saturated with sales in the first period, regardless of the leftover inventory of new units from the first period. However, as shown in Proposition 5, a greater availability of leftover inventory expands the region where only the new product is offered and shrinks the region in which the product line (i.e., both the new and the refurbished products) is offered. As we discuss subsequently, the possibility of carrying an inventory of new units from the first period into the second period – due to uncertainty in the defect rate – has implications for the firm’s time inconsistency problem.

Overall, Proposition 5 establishes the joint effect of $q_1$ and $I_1$ on the second-period equilibrium for any given defect rate $\alpha$. This is a crucial step before the firm’s first period decisions can be analyzed, because of strategic consumer behavior. As the firm and consumers make decisions in the first period with the defect rate being uncertain, Proposition 5 provides a predictive mechanism for the outcomes in the second period for any realization of the defect rate. We next discuss the first period decisions.
6.2 First Period Decisions

In the first period, the firm makes two decisions: the production quantity, denoted by \( Q_1 \), and the price of the new product \( p_1 \). Recall that we use \( \theta_m \) to denote the quality valuation of the marginal consumer who is indifferent between participating in the purchase of the new product in the first period and postponing the purchase to the second period, and \( q_m := 1 - \theta_m \) represents the quantity of consumers participating in the purchase in the first period. For given \( Q_1, p_1, q_m, \) and \( \alpha_i \), where \( i \in \{ L, H \} \), the expected net utility for a consumer of type \( \theta \in [\theta_m, 1] \) from buying the new product in the first period, is:

\[
U^\theta_{1i}(p_1, Q_1, q_m) = \begin{cases} 
  v_n \theta - p_1 - h \alpha_i & \text{if } Q_1 \geq (1 + \alpha_i) q_m, \\
  (v_n \theta - p_1 - h \alpha_i) \left( \frac{Q_1}{(1 + \alpha_i) q_m} \right) & \text{if } Q_1 < (1 + \alpha_i) q_m.
\end{cases}
\] (6.7)

Equivalently, \( U^\theta_{1i}(p_1, Q_1, q_m) = (v_n \theta - p_1 - h \alpha_i) \Phi_i(Q_1, q_m) \), where \( \Phi_i(Q_1, q_m) \) is the probability that the consumer is able to obtain a non-defective new unit. If the production quantity of the new product is sufficient to provide a non-defective new unit to all the consumers participating in the purchase (i.e., \( Q_1 \geq (1 + \alpha_i) q_m \)), then every participating consumer gets a non-defective new unit with certainty (i.e., \( \Phi_i(Q_1, q_m) = 1 \)).

On the other hand, if there is a stockout (i.e., \( Q_1 < (1 + \alpha_i) q_m \)), assuming proportional rationing by the firm in providing replacement units, a consumer is able to get a non-defective new unit with probability \( \Phi_i(Q_1, q_m) = \frac{Q_1}{(1 + \alpha_i) q_m} \).

For ease of exposition, we transform the first-period production quantity decision as: \( Q_1 := (1 + \alpha_Q) q_m \), wherein the transformed decision \( \alpha_Q \) captures the incremental quantity produced beyond the quantity of consumers participating in the purchase in the first period, to account for DPRs. For given first-period decisions \( \alpha_Q \) and \( p_1 \), the realized defect rate leads to one of the three scenarios summarized in Table 6.

In the first period, a consumer evaluates the expected discounted net utility from buying in the first period or postponing the purchase to the second period. Based on the above discussion, the expected net utility for a consumer of type \( \theta \) from buying the new product in the first period, \( U^\theta_{1e} := \sigma U^\theta_{1L} + (1 - \sigma) U^\theta_{1H} \), is given by:

\[
U^\theta_{1e}(p_1, \alpha_Q) = \begin{cases} 
  \sigma (v_n \theta - p_1 - h \alpha_L) + (1 - \sigma) (v_n \theta - p_1 - h \alpha_H) & \text{if } \alpha_Q > \alpha_H \\
  \sigma (v_n \theta - p_1 - h \alpha_L) + (1 - \sigma) (v_n \theta - p_1 - h \alpha_H) \left( \frac{1+\alpha_Q}{1+\alpha_L} \right) & \text{if } \alpha_L \leq \alpha_Q \leq \alpha_H \\
  \sigma (v_n \theta - p_1 - h \alpha_L) \left( \frac{1+\alpha_Q}{1+\alpha_L} \right) + (1 - \sigma) (v_n \theta - p_1 - h \alpha_H) \left( \frac{1+\alpha_Q}{1+\alpha_H} \right) & \text{if } \alpha_Q < \alpha_L.
\end{cases}
\] (6.8)

From the vantage point of the first period, the consumer’s second period utility is stochastic because the net utilities from buying the new product or the refurbished product in the second period depend on the realization of the defect rate. The consumer also considers the possibility that they may end up without any product in the second period if they postpone their purchase – either because a product is not offered or because the
consumer’s net utility from buying is negative. Note that even if the expected discounted net utility from postponing the purchase to the second period is greater than the expected net utility from buying in the first period, it may be optimal for the consumer to not purchase any product in the second period, after knowing the realization of the defect rate \( \alpha \) and the prices of the new and the refurbished products (if offered by the firm). Thus, the expected discounted net utility for a consumer of type \( \theta \) from postponing the purchase to the second period is 

\[
U_{1e}^{\theta_\theta} (p_1, \alpha_Q) = \sigma \left( \max \left\{ U_{2L}^{\theta_\theta}, U_{rL}^{\theta_\theta}, 0 \right\} \right) + (1 - \sigma) \left( \max \left\{ U_{2H}^{\theta_\theta}, U_{rH}^{\theta_\theta}, 0 \right\} \right).
\]

Thus, the quality valuation \( \theta_m \) of the marginal consumer, who is indifferent between participating in the purchase of the new product in the first period and postponing the purchase to the second period, satisfies:

\[
U_{1e}^{\theta_\theta} (p_1, \alpha_Q) = \sigma \left( \max \left\{ U_{2L}^{\theta_\theta}, U_{rL}^{\theta_\theta}, 0 \right\} \right) + (1 - \sigma) \left( \max \left\{ U_{2H}^{\theta_\theta}, U_{rH}^{\theta_\theta}, 0 \right\} \right) = \sigma \left( \max \left\{ U_{2L}^{\theta_\theta}, U_{rL}^{\theta_\theta}, 0 \right\} \right) + (1 - \sigma) \left( \max \left\{ U_{2H}^{\theta_\theta}, U_{rH}^{\theta_\theta}, 0 \right\} \right). \tag{6.10}
\]

Let \( \Pi(p_1, \alpha_Q) \) denote the total expected discounted profit of the firm from selling the new product in the first period, and the new product and/or the refurbished product in the second period if, in the first period, the firm chooses price \( p_1 \) and production quantity \( Q_1 = (1 + \alpha_Q) q_m \). Thus, the firm’s optimization problem in the first period is:

\[
\max_{\{p_1, \alpha_Q\}} \Pi(p_1, \alpha_Q) = \sigma q_m (p_1, \alpha_Q) \Phi_L (\alpha_Q, q_m (p_1, \alpha_Q)) (p_1 - (1 + \alpha_L) c_n) + (1 - \sigma) q_m (p_1, \alpha_Q) \Phi_H (\alpha_Q, q_m (p_1, \alpha_Q)) (p_1 - (1 + \alpha_H) c_n) + \rho [\sigma \Pi^*_2 (\alpha_L, q_{1L}, I_{1L}) + (1 - \sigma) \Pi^*_2 (\alpha_H, q_{1H}, I_{1H})] \tag{6.11}
\]

s.t. \( q_m (p_1, \alpha_Q) \geq 0, \alpha_Q \geq 0 \),

where \( \Pi^*_2 (\alpha_L, q_{1L}, I_{1L}) \) and \( \Pi^*_2 (\alpha_H, q_{1H}, I_{1H}) \) are the firm’s optimal second-period profits when the realized defect rates are low and high, respectively. \( q_{1L} := q_1 (\alpha_L, p_1, \alpha_Q), I_{1L} := I_1 (\alpha_L, p_1, \alpha_Q), q_{1H} := q_1 (\alpha_H, p_1, \alpha_Q) \), and \( I_{1H} := I_1 (\alpha_H, p_1, \alpha_Q) \) are the first-period sales quantities and leftover inventories when the realized defect rates are low and high, respectively. In Proposition 6 below, we show that \( \alpha_Q^* \) is bounded below and above by \( \alpha_L \) and \( \alpha_H \), respectively.

**Proposition 6.** The equilibrium production quantity \( Q_1^* = (1 + \alpha_Q^*) q_m \) is such that \( \alpha_L \leq \alpha_Q^* \leq \alpha_H \).  

**Proof.** See Appendix F.2. \( \square \)

Proposition 6 shows that equilibrium production quantity in the first period is such that: (i) If the realized defect rate is \( \alpha_L \), the firm would be able to fully meet demand (resulting in \( \Phi_L = 1 \), and leftover inventory \( I_1 \geq 0 \)); and, (ii) If the realized defect rate is \( \alpha_H \), the firm would face a stockout (resulting in proportional
rationing, $\Phi_H = \left(\frac{1+\alpha Q}{1+\alpha H}\right)$, and unmet demand $\chi_1 \geq 0)$. From equation (6.8) and Proposition 6, the expected net utility for the marginal consumer from participating in the purchase, is:

$$U^{\theta_m}_1(p_1, \alpha Q) = \sigma (\theta_m v_n - p_1 - h\alpha_L) + (1 - \sigma) (\theta_m v_n - p_1 - h\alpha_H) \left(\frac{1+\alpha Q}{1+\alpha H}\right).$$  \hspace{1cm} (6.12)

We derive the relationship $q_m(p_1, \alpha Q)$ using $q_m = 1 - \theta_m$ and equations (6.9), (6.10), and (6.12) (the expressions are omitted for brevity and are available upon request). Also, using Proposition 6, we can substitute $\Phi_L = 1, \Phi_H = \left(\frac{1+\alpha Q}{1+\alpha H}\right), q_{1L} = q_m, I_{1L} = (\alpha Q - \alpha L) q_m, q_{1H} = q_m \left(\frac{1+\alpha Q}{1+\alpha H}\right)$, and $I_{1H} = 0$ in (6.11).

### 6.3 Discussion

In this section, we examine the effect of uncertainty in the defect rate on the firm’s time inconsistency problem, including whether the firm’s profit could increase in the (expected) defect rate even when the defect rate is uncertain. As such, similar to the approach in the proof of Proposition 3(ii)(b), we focus our attention on the region where the firm optimally refurbishes all of the DPRs, so that the refurbished product is liable to displace the new product in the second period. Note that the firm’s optimization problem in (6.11) is intractable, partly because the objective function is not jointly concave in $p_1$ and $\alpha Q$. Therefore, we solve the problem numerically. Recall that the expected defect rate in the stochastic model is $\alpha_e := \sigma \alpha_L + (1 - \sigma) \alpha_H = \alpha$, facilitating comparisons with the deterministic model considered in the preceding sections. Also, to understand how the quantities, prices, and profits behave when the defect rate is uncertain, we vary $\alpha_e$ and $\Delta$ (keeping $\sigma$ fixed), noting that $\alpha_L = \alpha_e - (1 - \sigma) \Delta$ and $\alpha_H = \alpha_e + \sigma \Delta$.

We observe that even when the defect rate is uncertain, DPRs may serve as a commitment device by helping displace the new product in the second period, thereby mitigating the firm’s time inconsistency problem. Also, similar to our observation in §5.1, the firm’s expected profit could increase as the (expected) defect rate increases (see Figures 6(a) and (b), where the firm’s expected profit increases between the expected defect rates $\alpha_{e1}$ and $\alpha_{e2}$). However, under the stochastic model, the possibility of either leftover inventory of the new product from the first period due to overproduction, or unmet demand due to underproduction, results in a greater incentive for the firm to offer the new product in the second period compared to the deterministic model. As such, we observe that the rate of mitigation of the time inconsistency problem and the resulting non-monotonicity of firm profit are subdued under the stochastic model – more so when the uncertainty is greater (compare the non-monotonic portions of the dotted and solid curves corresponding to the stochastic and deterministic defect rate models, within and across Figures 6(a) and 6(b)). Additional discussion on how uncertainty in the defect rate impacts the firm’s equilibrium production and sales quantities is provided in Appendix F.3.
(a) Lower Uncertainty ($\Delta = 0.2\alpha_e$)  
(b) Higher Uncertainty ($\Delta = 0.4\alpha_e$)

Figure 6: Uncertain Defect Rate: Non-Monotonicity of Firm Profit with respect to the Defect Rate  
($c_n = 0.7$, $c_r = 0.1$, $v_n = 1.0$, $v_r = 0.9$, $\rho = 0.9$, $h = 0.05$, $k_r = 0.25$, $\sigma = 0.5$)

Tables 16, 17, and 18 in Appendix F.4 present equilibrium values of the production and sales quantities of the new product in the first period, the leftover inventory or unmet demand in the first period, the sales quantities of the new and the refurbished products in the second period, and expected firm profit for varying values of the expected defect rate $\alpha_e$ (and, in turn, $\alpha_L$ and $\alpha_H$) and the magnitude of uncertainty $\Delta$. Overall, these tables consistently illustrate the value of DPRs as a possible device for the firm to implicitly commit to limiting the production of new units in the second period, even when the defect rate is uncertain.

7. Conclusion

Quality issues in newly launched products, manifested in the form of defective product returns (DPRs), are a concern regardless of whether the firm is established or a newcomer (Heathman 2017, Smith et al. 1996). It is not uncommon for innovators to focus primarily on designing and launching new products and being underprepared with a strategy for DPRs (Smith et al. 1996). However, an important operational approach to managing DPRs comes in the form of refurbishing. Refurbishing allows firms to limit the negative impact of DPRs in two ways: directly, by recovering value from DPRs and, indirectly, through finer market segmentation in the future. However, with consumers being increasingly informed and strategic (or forward-looking), the refurbishing of DPRs also has a key downside: the future availability of a less expensive substitute may make consumers postpone their purchase, thus affecting earlier and more profitable sales of the new product. Therefore, it is unclear how DPRs and their potential refurbishing affect a firm’s intertemporal product line decisions in the presence of strategic consumers. We investigate this question in this paper.

If the firm were able to credibly commit to a future strategy, it would announce that the new product
will not be offered in the future, as a way to encourage consumers to buy the new product earlier at a higher price. However, such commitments are rarely credible in practice because of the firm’s incentive to offer the new product again in the future (Coase 1972). This inconsistent behavior of the firm over time (or time inconsistency) results in the firm’s profit being negatively affected if consumers are strategic (Bulow 1982). Our analysis shows that the refurbishing of DPRs can mitigate the firm’s time inconsistency problem by serving as an implicit and – to our knowledge – novel way to credibly commit that the new product will be scarce (or not available at all) in the future if consumers choose to wait. This allows the firm to charge a premium for the new product earlier. Furthermore, we observe the counterintuitive possibility that the firm’s profit may locally increase within a certain range of the defect rate.

We show that our qualitative findings are robust to several relaxations/extensions, including: a lower defect rate for new units in the future; a positive salvage value of unrefurbished DPRs; new consumers entering the market in the future; and, consumers opting for a refund instead of a replacement. Importantly, our qualitative findings persist even if the defect rate is uncertain when the product is introduced. However, under uncertainty in the defect rate, the firm’s ability to use DPRs as a commitment device is tempered: This is because of the possibility of either leftover inventory of the new product due to overproduction, or unmet demand due to underproduction. Both these possibilities induce the firm to offer the new product again in the future.

For managers who are wary of quality issues with innovative products, our analysis shows that the negative impact of returns of defective units may be overstated. While our results do not imply that some defects is better than no defects, they do indicate that opportunities to achieve marginal reductions in defect rates may not be worth the investment, and may even be counterproductive. In practice, eliminating defects altogether may be very expensive or even infeasible, especially in fast-paced industries such as consumer electronics, where a particular product may be produced and sold during a relatively short period of time for the firm to be able to completely eliminate defects. Our analysis also shows that efforts towards increasing the perceived quality of the refurbished product or decreasing the hassle cost for consumers may better serve the firm than efforts towards marginally improving the conformance quality of the new product. Consumer perceptions of the refurbished product can be influenced by communicating the rigor of refurbishing processes, sharing positive consumer reviews of refurbished units, and advocating the environmental benefits of purchasing refurbished products. Hassle costs for consumers could be reduced by helping them transfer user settings to the replacement units, and by efforts towards making the return experience more empathetic and pleasant.

Future research can further extend our work along several dimensions to understand the contexts in which our finding regarding the firm’s ability to use DPRs as a commitment device may be suppressed or enhanced: First, our analysis focuses on a market with strategic consumers to examine the potential role of DPRs as a commitment device. A more general formulation of the problem could allow for a mix of strategic and
non-strategic (or impatient) consumers. Second, because of our research focus, we assume the defect rate (including its uncertainty) to be common knowledge between the firm and consumers. While consumers may be able to infer defect rates from annual reports (for publicly traded firms; typically 10-Ks), and from product discussion forums and industry studies (such as those cited in this paper), the firm may have asymmetrically better information on the defect rate. Third, according to the 2011 Consumer Electronics Association study (Business Wire 2011), when returning a CE device, the most popular exchange consumers make is for the same model and same brand (38%), followed by a different model but the same brand (13%). 17% return a product for a different brand and 27% request some form of monetary compensation such as a refund or store credit. Although our analysis, including the extensions, treats the majority (65%) of these instances, the consideration of substitutable products offered by the same or another firm will extend the coverage of the return instances in practice. Finally, factors such as the trade-off between shortening the time to market and improving product quality, internal and external competition from products with lower defect rates, and longer-term interactions between defect rates and brand reputation could be considered in future work.

References


Reddit (2016). What is your experience with refurbished products? [https://www.reddit.com/r/Frugal/comments/4b4w7y/what_is_your_experience_with_refurbished_products/](https://www.reddit.com/r/Frugal/comments/4b4w7y/what_is_your_experience_with_refurbished_products/). March 19.


Tables

<table>
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<tr>
<th>Product Strategy Notation</th>
<th>Firm’s Decisions</th>
<th>1st Period</th>
<th>2nd Period</th>
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<td></td>
<td></td>
<td>New Product</td>
<td>New Product</td>
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<td>√</td>
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<tr>
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<td>×</td>
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Table 1: Firm’s Product Strategy Space

<table>
<thead>
<tr>
<th>Strategy</th>
<th>( q^<em>_N (q₁) ) and ( q^</em>_q (q₁) )</th>
<th>( p^<em>_N (q₁) ) and ( p^</em>_q (q₁) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNRₐ</td>
<td>[ q^<em>_N (q₁) = \frac{1}{2} \left( \frac{\mu_n - \mu_r}{v_n - v_r} - q₁ \right); ] [ q^</em>_q (q₁) = \frac{1}{2} \left( \frac{\mu_n - \mu_r}{v_n - v_r} - q₁ \right); ]</td>
<td>[ p^<em>_N (q₁) = v_n (1 - q₁) - h_N q₁ - \frac{\mu_n - v_n q₁}{2}; ] [ p^</em>_q (q₁) = v_r (1 - q₁) - h_q q₁ - \frac{\mu_r - v_r q₁}{2}; ]</td>
</tr>
<tr>
<td>NNRₐₐ</td>
<td>[ q^<em>_N (q₁) = \frac{1}{2} \left( \frac{\mu_n - \mu_r}{v_n - v_r} - \frac{\alpha q_1 v_1}{v_n (1 + k_r \alpha)} \right); ] [ q^</em>_q (q₁) = \frac{1}{2} \left( \frac{\mu_n - \mu_r}{v_n - v_r} - \frac{\alpha q_1 v_1}{v_n (1 + k_r \alpha)} \right); ]</td>
<td>[ p^<em>_N (q₁) = v_n (1 - q₁) - h_N q₁ - \frac{\mu_n - v_n q₁}{2}; ] [ p^</em>_q (q₁) = v_r (1 - q₁) - h_q q₁ - \frac{\mu_r - v_r q₁}{2}; ]</td>
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<td>[ q^<em>_N (q₁) = 0; ] [ q^</em>_q (q₁) = \frac{1}{2} \left( \frac{\mu_r}{v_r} - q₁ \right); ]</td>
<td>[ p^*_N (q₁) = v_r (1 - q₁) - h_q q₁ - \frac{\mu_r - v_r q₁}{2}; ]</td>
</tr>
<tr>
<td>NNRₐₐₐₐ</td>
<td>[ q^<em>_N (q₁) = 0; ] [ q^</em>_q (q₁) = \frac{\alpha q_1 v_1}{1 + k_r \alpha}; ]</td>
<td>[ p^*_N (q₁) = v_r (1 - q₁) - h_q q₁ - \frac{\alpha q_1 v_1}{1 + k_r \alpha}; ]</td>
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<tr>
<td>NNRₐₐₐₐₐ</td>
<td>[ q^<em>_N (q₁) = \frac{1}{2} \left( \frac{\mu_n}{v_n} - q₁ \right); ] [ q^</em>_q (q₁) = 0; ]</td>
<td>[ p^*_N (q₁) = v_n (1 - q₁) - h_N q₁ - \frac{\mu_n - v_n q₁}{2}; ]</td>
</tr>
<tr>
<td>NNRₐₐₐₐₐₐ</td>
<td>[ q^<em>_N (q₁) = \frac{\mu_n}{2v_n}; ] [ q^</em>_q (q₁) = 0; ]</td>
<td>[ p^*_N (q₁) = v_n - h_N q₁ - \frac{\mu_n}{2}; ]</td>
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Table 2: Optimal Sales Quantities and Prices in the Second Period (given \( q₁ \)
<table>
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<tr>
<th>Strategy</th>
<th>Conditions for Optimal Strategies in the Second Period (given $q$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NMR_4$</td>
<td>$\phi_{NMR_4}(q) = \frac{1}{1 + k_3} \left( \frac{v_n - \alpha}{\mu_n} \right) &gt; 0$</td>
</tr>
<tr>
<td>$NMR_3$</td>
<td>$\phi_{NMR_3}(q) = \frac{1}{1 + k_3} \left( \frac{v_n - \alpha}{\mu_n} \right) &gt; 0$</td>
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<tr>
<td>$NMR_2$</td>
<td>$\phi_{NMR_2}(q) = \frac{1}{1 + k_3} \left( \frac{v_n - \alpha}{\mu_n} \right) &gt; 0$</td>
</tr>
<tr>
<td>$NMR_1$</td>
<td>$\phi_{NMR_1}(q) = \frac{1}{1 + k_3} \left( \frac{v_n - \alpha}{\mu_n} \right) &gt; 0$</td>
</tr>
</tbody>
</table>

Table 3: Conditions for Optimal Strategies in the Second Period (given $q$)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Conditions when the Strategy is Optimal in the Second Period</th>
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<tbody>
<tr>
<td>$NMR_4$</td>
<td>$\phi_{NMR_4}(q) &gt; 0$, $\phi_{NMR_4}(q) &gt; 0$</td>
</tr>
<tr>
<td>$NMR_3$</td>
<td>$\phi_{NMR_3}(q) &gt; 0$, $\phi_{NMR_3}(q) &gt; 0$</td>
</tr>
<tr>
<td>$NMR_2$</td>
<td>$\phi_{NMR_2}(q) &gt; 0$, $\phi_{NMR_2}(q) &gt; 0$</td>
</tr>
<tr>
<td>$NMR_1$</td>
<td>$\phi_{NMR_1}(q) &gt; 0$, $\phi_{NMR_1}(q) &gt; 0$</td>
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Table 4: Equilibrium Sales Quantity and Price of the New Product in the First Period

<table>
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<th>Strategy</th>
<th>Equilibrium Sales Quantity and Price of the New Product in the First Period</th>
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<tbody>
<tr>
<td>$NMR_4$</td>
<td>$q^* = \max \left{ \frac{v_n - \alpha}{\mu_n} - \frac{\mu_n}{\mu_n} \left( \frac{v_n - \alpha}{\mu_n} \right) \right}$</td>
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<tr>
<td>$NMR_3$</td>
<td>$q^* = \max \left{ \frac{v_n - \alpha}{\mu_n} - \frac{\mu_n}{\mu_n} \left( \frac{v_n - \alpha}{\mu_n} \right) \right}$</td>
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<td>$NMR_2$</td>
<td>$q^* = \max \left{ \frac{v_n - \alpha}{\mu_n} - \frac{\mu_n}{\mu_n} \left( \frac{v_n - \alpha}{\mu_n} \right) \right}$</td>
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<tr>
<td>$NMR_1$</td>
<td>$q^* = \max \left{ \frac{v_n - \alpha}{\mu_n} - \frac{\mu_n}{\mu_n} \left( \frac{v_n - \alpha}{\mu_n} \right) \right}$</td>
</tr>
</tbody>
</table>
NNRS  \[ \frac{\alpha q_{NNRS}}{1 + k_r \alpha} > q_{NNRS} \left( \frac{q_1}{q_{NNRS}} \right) > 0; q_{NNRS} \left( \frac{q_1}{q_{NNRS}} \right) > 0; \]
\[ \Pi_{NNRS} \geq \max \{ \Pi_{NNRS, N^NRA, N^N\emptyset S, N^N\emptyset A, N^N\emptyset} \}. \]

NNRA  \[ q_{NNRA} \left( \frac{q_1}{q_{NNRA}} \right) \geq \frac{\alpha q_{NNRA}}{1 + k_r \alpha} > 0; q_{NNRA} \left( \frac{q_1}{q_{NNRA}} \right) > 0; q_{NNRA} \left( \frac{q_1}{q_{NNRA}} \right) > 0; \]
\[ \Pi_{NNRA} \geq \max \{ \Pi_{NNRS, N^NRA, N^N\emptyset S, N^N\emptyset A, N^N\emptyset} \}. \]

N\emptyset S  \[ \frac{\alpha q_{N\emptyset S}}{1 + k_r \alpha} > q_{N\emptyset S} \left( \frac{q_1}{q_{N\emptyset S}} \right) > 0; q_{N\emptyset S} \left( \frac{q_1}{q_{N\emptyset S}} \right) \leq 0; \]
\[ q_{N\emptyset S} \left( \frac{q_1}{q_{N\emptyset S}} \right) \leq 0; \]
\[ \Pi_{N\emptyset S} \geq \max \{ \Pi_{NNRS, N^NRA, N^N\emptyset S, N^N\emptyset A, N^N\emptyset} \}. \]

N\emptyset A  \[ q_{N\emptyset A} \left( \frac{q_1}{q_{N\emptyset A}} \right) \geq \frac{\alpha q_{N\emptyset A}}{1 + k_r \alpha} > 0; q_{N\emptyset A} \left( \frac{q_1}{q_{N\emptyset A}} \right) \leq 0; q_{N\emptyset A} \left( \frac{q_1}{q_{N\emptyset A}} \right) > 0; \]
\[ \Pi_{N\emptyset A} \geq \max \{ \Pi_{NNRS, N^NRA, N^N\emptyset S, N^N\emptyset A, N^N\emptyset} \}. \]

N\emptyset  \[ q_{N\emptyset} > 0; q_{N\emptyset} \left( \frac{q_1}{q_{N\emptyset}} \right) > 0; q_{N\emptyset} \left( \frac{q_1}{q_{N\emptyset}} \right) \leq 0; \]
\[ \Pi_{N\emptyset} \geq \max \{ \Pi_{NNRS, N^NRA, N^N\emptyset S, N^N\emptyset A, N^N\emptyset} \}. \]

Table 5: Conditions for Equilibrium Strategies

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Realized Defect Rate</th>
<th>First Period Production ((Q_1))</th>
<th>First Period Sales ((q_1))</th>
<th>Leftover Inventory ((I_1))</th>
<th>Unmet Demand ((\chi_1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overproduction</td>
<td>(\alpha_i &lt; \alpha_Q)</td>
<td>(Q_1 &gt; (1 + \alpha_i) q_m)</td>
<td>(q_m)</td>
<td>((\alpha_Q - \alpha_i) q_m)</td>
<td>0</td>
</tr>
<tr>
<td>Balanced Production</td>
<td>(\alpha_i = \alpha_Q)</td>
<td>(Q_1 = (1 + \alpha_i) q_m)</td>
<td>(q_m)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Underproduction</td>
<td>(\alpha_i &gt; \alpha_Q)</td>
<td>(Q_1 &lt; (1 + \alpha_i) q_m)</td>
<td>(\left( \frac{1 + \alpha_Q}{1 + \alpha_i} \right) q_m)</td>
<td>0</td>
<td>(\left( \frac{\alpha_i - \alpha_Q}{1 + \alpha_i} \right) q_m)</td>
</tr>
</tbody>
</table>

Table 6: First-Period Scenarios when the Defect Rate is Uncertain
Online Appendix A for:

Intertemporal Product Management with Strategic Consumers:
The Value of Defective Product Returns

(Appendices B through F are available at http://dx.doi.org/10.2139/ssrn.2511424)

A. Proofs for §3, §4, and §5

A.1 Statement and Proof of Lemma 1 (§3)

Lemma 1. Since consumers are distributed in a continuum, in order to realize a sales quantity \( q \) of the new [refurbished] product, the firm should produce a quantity \( (1 + \alpha)q \) [refurbish a quantity \( (1 + k_r \alpha)q \)].

Proof. Since a fraction \( \alpha' \) of new units are defective, the number of transactions \( (Y_i) \) until a consumer \( i \) obtains a non-defective new unit has a geometric distribution with parameter \( p = 1 - \alpha' \) (probability of receiving a non-defective unit in a transaction), mean \( \frac{1}{p} = \frac{1}{1-\alpha'} \), and variance \( \frac{1-p^2}{p^2} = \frac{\alpha'}{(1-\alpha')^2} \). Suppose there are \( N \) consumers who purchase the product. Let \( Y := \frac{1}{N} \sum_{i=1}^{N} Y_i \) denote the average number of transactions needed per purchasing consumer in order to provide each of them with a non-defective unit.

Therefore, \( E(Y) = \frac{1}{N} \sum_{i=1}^{N} E(Y_i) = \frac{N \cdot E(Y_i)}{N} = \frac{1}{1-\alpha} = 1 + \alpha \), since we define \( \alpha = \frac{\alpha'}{1-\alpha} \). Also, \( \text{Var}(Y) = \frac{1}{N^2} \sum_{i=1}^{N} \text{Var}(Y_i) = \frac{N \cdot \text{Var}(Y_i)}{N^2} = \frac{1}{N} \frac{\alpha'}{(1-\alpha')^2} \). However, since we consider a continuum of consumers (i.e., \( N \to \infty \)), \( \lim_{N \to \infty} \text{Var}(Y) = 0 \). Therefore, in order to realize a sales quantity \( q \), the firm should produce a quantity equal to \( q(1 + \alpha) \). The proof is similar for the refurbished product, replacing \( \alpha \) with \( k_r \alpha \).

A.2 Proof of Proposition 1 (§4.2)

We substitute \( q_2(q_1, P) \) and \( q_r(q_1, P) \) from (4.2) in the firm’s problem (4.3) and solve for \( p_2 \) and \( p_r \) (note that \( \Pi_2 \) is jointly concave in \( p_2 \) and \( p_r \)). Table 2 summarizes the optimal second-period prices and sales quantities (as functions of \( q_1 \)) for the different product strategies.

We derive the conditions (summarized in Table 3) under which the different product strategies are optimal in the second period. Note that the thresholds \( \tilde{\alpha} := \frac{c_r v_n - c_n v_r}{v_r (c_n + h) - v_n (c_r + h) k_r} \), \( \tilde{q}_1 := \min \left\{ \frac{\mu_n - \mu_r}{v_n - v_r}, \frac{\mu_n (1 + k_r \alpha)}{v_n (1 + k_r \alpha) + 2 \alpha v_r} \right\} \), and \( \tilde{q}_1 := \max \left\{ \frac{\mu_n}{v_n}, \frac{\mu_r}{v_r} \right\} \) referred to below, arise from comparing the profits between product strategies, where \( \mu_n \) and \( \mu_r \) are as defined in §4.1. Also note that \( \alpha \leq \tilde{\alpha} \iff \frac{\mu_n}{v_n} \geq \frac{\mu_r}{v_r} \).

(i) No (\( q_2^* = 0, q_r^* = 0 \)): When \( \alpha \leq \tilde{\alpha} \), the first-order conditions (FOCs) of the firm’s second-period problem (4.3) yield \( q_2(q_1) \leq 0 \) and \( q_r(q_1) \leq 0 \) if \( q_1 \geq \frac{\mu_n}{v_n} \). When \( \alpha > \tilde{\alpha} \), the FOCs yield \( q_2(q_1) \leq 0 \) and \( q_r(q_1) \leq 0 \) if \( q_1 \geq \frac{\mu_r}{v_r} \). Thus, if \( q_1 \geq \tilde{q}_1 \), we have \( q_2^* = 0 \) and \( q_r^* = 0 \), i.e., the firm offers neither of the products in the second period.
(ii) **New Only** \((q_2^* > 0, q_r^* = 0)\): When \(\alpha \leq \bar{\alpha}\), the FOCs yield \(q_2(q_1) = \frac{(\mu_n - v_n q_1)}{2v_n} > 0\) and \(q_r(q_1) \leq 0\) if \(q_1 \leq \bar{q}_1\). Thus, if \(\alpha \leq \bar{\alpha}\) and \(q_1 < \bar{q}_1\), we have \(q_2^* > 0\) and \(q_r^* = 0\), i.e., the firm offers only the new product in the second period.

(iii) **Refurbished Only** \((q_2^* = 0, q_r^* > 0)\): When \(\alpha > \bar{\alpha}\) and \(\alpha q_1 > (1 + k_r \alpha) q_r(q_1)\), the FOCs yield \(q_r(q_1) = \frac{\mu_n - v_n q_1}{v_n^2} > 0\) and \(q_2(q_1) \leq 0\), if \(\max \left\{ \frac{\mu_n - v_n q_1}{v_n^2}, \frac{\mu_n (1 + k_r \alpha)}{v_n (1 + k_r \alpha + 2 \alpha)} \right\} \leq q_1 < \frac{\mu_n}{v_n} \). When \(\alpha > \bar{\alpha}\) and \(\alpha q_1 = (1 + k_r \alpha) q_r(q_1)\), the FOCs yield \(q_r(q_1) = \frac{\alpha q_1}{1 + k_r \alpha}\) and \(q_2(q_1) \leq 0\) if \(\frac{\mu_n (1 + k_r \alpha)}{v_n (1 + k_r \alpha + 2 \alpha)} \leq q_1 \leq \frac{\mu_n (1 + k_r \alpha)}{v_n (1 + k_r \alpha + 2 \alpha)}\). Thus, if \(\alpha > \bar{\alpha}\) and \(q_1 \leq q_1 < \bar{q}_1\), we have \(q_2^* = 0\) and \(q_r^* > 0\), i.e., the firm offers only the refurbished product in the second period.

(iv) **Product Line** \((q_2^* > 0, q_r^* > 0)\): When \(\alpha > \bar{\alpha}\) and \(\alpha q_1 > (1 + k_r \alpha) q_r(q_1)\), the FOCs yield \(q_2(q_1) = \frac{1}{2} \left( \frac{\mu_n - v_n q_1}{v_n - q_1} \right) > 0\) and \(q_r(q_1) = \frac{v_n (1 + (1 + \alpha) \alpha) - v_n (1 + k_r \alpha) + h_{k_r \alpha}}{2 v_n (v_n - q_1)} \geq 0\) if \(\left( \frac{1 + k_r \alpha}{\alpha} \right) \left( \frac{v_n \mu_n - v_n \mu_n}{2 v_n (v_n - q_1)} \right) < q_1 < \frac{\mu_n - v_n q_1}{v_n - q_1} \). When \(\alpha > \bar{\alpha}\) and \(\alpha q_1 = (1 + k_r \alpha) q_r(q_1)\), the FOCs yield \(q_2(q_1) = \frac{\mu_n - v_n q_1}{v_n^2} < q_1 < \frac{\alpha q_1}{1 + k_r \alpha}\) if \(q_1 < \min \left\{ \frac{\mu_n (1 + k_r \alpha)}{v_n (1 + k_r \alpha + 2 \alpha)}, \frac{\alpha q_1}{1 + k_r \alpha} \left( \frac{v_n \mu_n - v_n \mu_n}{2 v_n (v_n - q_1)} \right) \right\} \). Thus, if \(\alpha > \bar{\alpha}\) and \(q_1 < \bar{q}_1\), we have \(q_2^* > 0\) and \(q_r^* > 0\), i.e., the firm offers the product line in the second period.

\[
\begin{align*}
\text{Proof.} & \quad \text{We first show that Strategy } N\emptyset \text{ cannot be optimal because it is strictly dominated by Strategy } N N\emptyset. \\
\text{Then we show that Strategy } N\emptyset & \text{ is suboptimal because it results in non-positive firm profit.} \\
\text{For } N N\emptyset, \text{ in which } q_r = 0, \text{ the equilibrium solutions are } q_1^* = \frac{2(1 - \rho) \mu_n}{v_n (1 - 3 \rho)}, \quad q_2^* = \frac{\mu_n (2 - \rho)}{2 v_n (1 - 3 \rho)}, \quad \text{and} \\
\Pi_{NN\emptyset}^* & = \frac{(\mu_n (2 - \rho))^2}{4 v_n (1 - 3 \rho)}. \quad \text{For Strategy } \emptyset \emptyset, \text{ in which } q_1 = q_r = 0, \text{ we have } q_2^* = \frac{\mu_n}{2 v_n} \quad \text{and} \quad \Pi_{\emptyset \emptyset}^* = \frac{\rho \mu_n^2}{4 v_n} < \Pi_{NN\emptyset}^*. \quad \text{Therefore, Strategy } \emptyset \emptyset \text{ is strictly dominated by Strategy } N N\emptyset \text{ and cannot be optimal.} \\
\text{Next, suppose Strategy } N\emptyset \text{ is optimal, wherein } q_2 = q_r = 0. \text{ In the first period, the marginal consumer who is indifferent between purchasing and not purchasing the new product is located at } \theta_m := 1 - q_1^* \text{ and obtains zero net utility from the purchase (i.e., } v_n \theta_m - h_{\alpha} - p_1^* = 0, \text{ or } p_1^* = v_n \theta_m - h_{\alpha}). \end{align*}
\]

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does not offer the new product either, it would have to be true that $v_n h_m - h \alpha \leq (1 + \alpha) c_n \implies p^*_r \leq (1 + \alpha) c_n$, implying that the firm does not make a positive profit. Therefore, Strategy $N \emptyset \emptyset$ is suboptimal.

From Lemma 2 and Proposition 1, $q_1 < \tilde{q}_1$ must hold, and the plausible equilibrium product strategies are listed in Table 4 (note that Table 2 provides the optimal second-period quantities and prices for the strategies listed in Table 4). The conditions (summarized in Table 5) for the different equilibrium second-period product strategies listed in the statement of Proposition 2 are derived below:

(i) New Only ($N \emptyset \emptyset$): Let $v^N_r(\alpha) = \frac{v_n (c_r (1 + k_r \alpha) + h k_r \alpha)}{(1 + \alpha) c_n + h \alpha}$. Observe that $v_r \leq v^N_r \iff \alpha \leq \bar{\alpha} = \frac{c_r v_n - c_n v_r}{v_r c_n + h v_n (c_r + h k_r) k_r}$.

Using Proposition 1(ii), when $\alpha \leq \bar{\alpha}$ and $q_1 < \tilde{q}_1$, the firm offers only the new product in the second period.

(ii) Product Line ($NNR_S$ and $NNR_A$): The firm offers both the new and the refurbished products in the second period, refurbishing some of the DPRs (i.e., Strategy $NNR_S$) if $c_r > \bar{c}_r$ (using the condition $(1 + k_r \alpha) q^*_r < \alpha q^*_1$) and $v_r < v^R_r$ (using the condition $q^*_2 > 0$), where:

\[
\bar{c}_r := v_r \left( (1 + \alpha) c_n + h \alpha \right) - v_n h k_r \alpha - \frac{4 (1 - \rho) \alpha v_r (v_n - v_r) \mu_n}{v_n (1 + k_r \alpha)}
\]

\[
v^R_r := v_n - \frac{v_n (4 - 3 \rho) [(c_r (1 + k_r \alpha) + h k_r \alpha) - (c_r (1 + k_r \alpha) + h k_r \alpha)]}{v_n (4 - 3 \rho) - 2 (1 - \rho) \mu_n}.
\]

The firm offers both the new and the refurbished products in the second period, refurbishing all of the DPRs (i.e., Strategy $NNR_A$) if $c_r \leq \bar{c}_r$ (using the condition $(1 + k_r \alpha) q^*_r = \alpha q^*_1$) and the following condition holds (so that $q^*_2 > 0$):

\[
2 (v_n (1 + k_r \alpha) + 2 \alpha v_r) [(1 - \rho) (1 + k_r \alpha) v_n \mu_n - \rho \alpha (\mu_n v_r - \mu_r v_n)] \mu_n \left[ v^2_n (4 - 3 \rho) (1 + k_r \alpha)^2 + 4 \rho \alpha^2 v_r (v_n - v_r) \right] < 1
\]

(A.13)

The roots of (A.13) in $v_r$ at equality are $X + \sqrt{Y}$ and $X - \sqrt{Y}$, where

\[
X = \frac{\mu_n [(3 \rho - 2) (1 + k_r \alpha) + 2 \alpha \rho] + 2 \rho \alpha (c_r (1 + k_r \alpha) + h k_r \alpha) - \rho v_n (1 + k_r \alpha) \mu_n}{4 \rho \alpha^2}
\]

\[
Y = \frac{v_n (1 + k_r \alpha) [\mu_n (2 - \rho) (1 + k_r \alpha) + 2 \rho \alpha (c_r (1 + k_r \alpha) + h k_r \alpha)]}{4 \rho \alpha^2} + X^2
\]

Since $X^2 - Y < 0$, the only non-negative root is $v^R_r := X + \sqrt{Y}$, and (A.13) holds when $v_r < v^R_r$.

Let $v^R := \max\{v^R_r, v^R_r\}$. We can show that $v^R_r \geq v^R_r \iff c_r \leq \bar{c}_r$. Therefore, when $v^R_r < v^R_r$ (implying $c_r > \bar{c}_r$), the firm offers the product line, refurbishing some of the DPRs (i.e., Strategy $NNR_S$) if $v_r < v^R_r = v^R_r$. On the other hand, when $v^R_r \geq v^R_r$ (implying $c_r \leq \bar{c}_r$), the firm offers the product line, refurbishing all of the DPRs (i.e., Strategy $NNR_A$) if $v_r < v^R_r = v^R_r$. From part (i) above, only the new product is offered if $v_r \leq v^N_r$. Thus, the firm offers the product line in the second period if $v^N_r < v_r < v^R_r$.

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(iii) Refurbished Only ($N\emptyset R_S$ and $N\emptyset R_A$): When $v_r > v^R_r$, we have $q_2^* = 0$ and $q_3^* > 0$. Thus, the firm offers only the refurbished product in the second period. The firm refurbishes some of the DPRs (i.e., Strategy $N\emptyset R_S$) if $c_r > \hat{c}_r$, and all of the DPRs (i.e., Strategy $N\emptyset R_A$) if $c_r \leq \hat{c}_r$, where:

\[
\hat{c}_r := \begin{cases} 
  c_{r1}, & \text{if } v^R_r \leq v_r < v^{R1}_r; \\
  c_{r2}, & \text{if } v^{R1}_r \leq v_r < v^{R2}_r; \\
  c_{r3}, & \text{if } v_r \geq v^{R2}_r.
\end{cases}
\]

1. $c_{r1}$ arises from equating the equilibrium profits between strategies $N\emptyset R_S$ and $N\emptyset R_A$, and are given by the expressions below:

\[
c_{r1} = \frac{v_r c_n (1 + \alpha) (1 + k_r \alpha + 2\alpha) - \alpha (2v_r (v_n - v_0) + h (v_n k_r (1 + k_r \alpha) - v_r (1 - k_r \alpha + 2\alpha)))}{(1 + k_r \alpha) (v_n (1 + k_r \alpha) + 2av_r)}.
\]

2. $c_{r2}$ and $c_{r3}$ arise from equating the equilibrium profits between strategies $N\emptyset R_S$ and $N\emptyset R_A$, and are given by the expressions below:

\[
c_{r2} = \frac{v_r - hk_r \alpha}{1 + k_r \alpha} - \frac{\rho \alpha v^2 \mu_n (5 (1 + k_r \alpha) + 4\alpha) + 4v_n v_r \mu_n (1 + k_r \alpha) (1 + k_r \alpha + \alpha)}{(1 + k_r \alpha) ((2v_n (1 + k_r \alpha))^2 + \rho \mu_n v_r (1 + k_r \alpha) (1 + k_r \alpha + 8\alpha) + 2\alpha \rho^2 v_r^2 (1 + k_r \alpha + 2\alpha))}
\]

\[
+ \frac{v_r \mu_n (4v_n - 3\rho v_r) (v_n (1 + k_r \alpha))^2 + \rho \mu_n v_r (1 + k_r \alpha) (1 + k_r \alpha + 8\alpha) + 2\alpha \rho^2 v_r^2 (1 + k_r \alpha + 2\alpha))}{(2v_n (1 + k_r \alpha))^2 + \rho \mu_n v_r (1 + k_r \alpha) (1 + k_r \alpha + 8\alpha) + 2\alpha \rho^2 v_r^2 (1 + k_r \alpha + 2\alpha)}.
\]

3. $v^R_r := v_r : c_{r1} = c_{r2}$, and $v^{R2}_r := v_r : c_{r2} = c_{r3}$.

\[\square\]

### A.4 Proof of Proposition 3 (§5.1)

Proposition 3(i): In the commitment scenario, the firm’s profit ($\Pi_C$) decreases monotonically in the defect rate $\alpha$. We derive the equilibrium solutions for the commitment scenario (denoted by superscript $C$) in Appendix B. Substituting the equilibrium prices and sales quantities for each of the strategies ($N\emptyset$, $N\emptyset R_S$, and $N\emptyset R_A$) from Table 8 in Appendix B into the firm’s profit function in (B.20), we obtain:

(a) Strategy $N\emptyset$:

\[
\frac{d\Pi_{N\emptyset}^C}{d\alpha} = -(c_n + h) q_1^C < 0
\]

(b) Strategy $N\emptyset R_S$:

\[
\frac{d\Pi_{N\emptyset R_S}^C}{d\alpha} = -(c_n + h) q_1^C - \rho k_r (c_r + h) q_2^C < 0.
\]
(c) **Strategy N∅RA:**

\[
\frac{d\Pi^C_{N∅RA}}{d\alpha} = -\left(1 + k_r \alpha \right) \left( v_n (1 + k_r \alpha)^2 + \rho v_r (1 + 2k_r \alpha) \right) \left( c_n (1 + k_r \alpha) \left( v_n (1 + k_r \alpha)^2 - \rho v_r \right) + \rho v_r \alpha (v_n - \rho v_r) + v_n (1 + k_r \alpha) (h + \rho c_r + k_r \alpha) (h + \rho (c_r + h)) + \rho \alpha v_r (h + \rho c_r + k_r \alpha) (3 + \alpha + 2k_r \alpha) (c_n + h + \rho (c_r + h)) \right) < 0.
\]

Thus, in the commitment scenario, the firm’s profit always decreases in the defect rate.

**Proposition 3(ii):** We first show $\Pi^* \leq \Pi^C$ by following the logic in Bulow (1986). Let $S$ be the set of feasible solutions $\{p_1, p_2, p_r\}$ in our setting and $S^C$ be the set of feasible solutions in the commitment scenario. Note that, in the commitment scenario, the firm sets the prices for both periods at the beginning of the first period. Therefore, in the commitment scenario, the firm’s problem is solved as a one-shot game between the firm and consumers (see (B.20) in Appendix B). In contrast, in our setting, where the firm cannot commit to second-period decisions, the firm sets the prices for the second period at the beginning of the second period (see (4.3) in §4.1). Therefore, the solution for the complete two-period problem (4.5) in our setting, unlike in the commitment scenario, must be a subgame perfect Nash equilibrium. Clearly, $S \subseteq S^C$. Therefore, $\Pi^* \leq \Pi^C$.

(a) **There exists a threshold defect rate, $\alpha_c$, beyond which firm profit in our setting equals that in the commitment scenario** (i.e., $\Pi^* = \Pi^C$).

From the equilibrium sales quantities for the commitment scenario in Table 8 in Appendix B, we obtain the following conditions for the different product strategies to be feasible in equilibrium: $\mu_n \geq \mu_r \left( \frac{v_n}{v_r} \right) > 0$ for Strategy $N∅∅$; $\mu_n > \rho \mu_r > 0$ for Strategy $N∅R_S$; and, $\mu_n > -\mu_r \left( \frac{\rho \alpha}{1 + k_r \alpha} \right)$ for Strategy $N∅RA$.

In our setting, there are five plausible equilibrium product strategies (see Table 4). From the equilibrium sales quantities in Table 4, we obtain the following conditions for the different product strategies to be feasible in equilibrium: $\mu_n \geq \mu_r \left( \frac{v_n}{v_r} \right) > 0$ for Strategy $NN∅∅$; $\mu_n > \mu_r > 0$ for Strategy $NNR_S$; $\mu_n > \rho \mu_r > 0$ for Strategy $NN∅R_S$; $\mu_n > 0$ for Strategy $NNRA$; and, $\mu_n > -\mu_r \left( \frac{\rho \alpha}{1 + k_r \alpha} \right)$ for Strategy $NN∅RA$.

Note that $-\partial \mu_n / \partial \alpha > -\partial \mu_r / \partial \alpha > 0$, and $\mu_r > 0$. Thus, when $-\mu_r \left( \frac{\rho \alpha}{1 + k_r \alpha} \right) < \mu_n \leq 0$ or, equivalently, when if $\alpha \geq \left( \frac{\rho \alpha}{c_n + h} \right) := \alpha_c$, Strategy $N∅RA$ is the equilibrium product strategy in both the commitment scenario and in our setting, and the resulting profits are identical, i.e., $\Pi^* = \Pi^C$. In other words, when $\alpha \geq \alpha_c$, it is optimal for the firm to not offer the new product in the second period in our setting as well.

(b) **There exists $\hat{v}_r$ such that firm profit in our setting ($\Pi^*$) is non-monotonic in the the defect rate $\alpha$ when $\hat{v}_r < v_r < v_n$:***
We consider the situation where \(v_r\) is sufficiently large so that the firm refurbishes all of the DPRs (i.e., \(\alpha q_1^r = (1 + k_r \alpha) q^*_r > 0\) holds, and either Strategy \(N\overline{N}R_A\) or Strategy \(N\emptyset R_A\) is feasible in equilibrium). This threshold for \(v_r\), denoted by \(\hat{v}_{r1}\), can be derived based on the optimal/equilibrium sales quantities in Tables 2 and 4 (we omit the expression of \(\hat{v}_{r1}\) for brevity). When \(v_r > \hat{v}_{r1}\), we show that:

\[(b1)\] The firm’s profit decreases in \(\alpha\) at \(\alpha = 0^+\); and,

\[(b2)\] There exist \(\hat{v}_{r2}\) and \(\alpha_s\) such that the firm’s profit increases in \(\alpha\) at \(\alpha_s\) for \(v_r > \hat{v}_{r2}\).

\[(b1)\] When \(v_r > \hat{v}_{r1}\) and \(\alpha \to 0^+\), Strategy \(\overline{N}NR_A\) is the equilibrium strategy (i.e., \(\alpha q_1^r = (1 + k_r \alpha) q^*_r > 0\) and \(q^*_2 > 0\), and we have:

\[
\Pi^*_\overline{N}NR_A\bigg|_{\alpha \to 0^+} = \frac{(2 - \rho)^2 (v_n - c_n)^2}{4v_n (4 - 3\rho)},
\]

\[
\frac{d\Pi^*_\overline{N}NR_A}{d\alpha}\bigg|_{\alpha \to 0^+} = -\frac{(v_n - c_n) \left(c_n (4 (1 - \rho) (v_n - \rho v_r) + \rho^2 v_n) + 4c_n v_n \rho (1 - \rho) + h v_n (2 - \rho)^2\right)}{2v_n^2 (4 - 3\rho)} < 0.
\]

\[(b2)\] We know from part (a) above, that the firm’s profit in our setting equals the profit in the commitment scenario when \(\alpha \geq \alpha_c\) because it is optimal for the firm to not offer the new product in the second period in our setting as well. Therefore, we focus on the value of \(\alpha\) beyond which the firm can implicitly commit that \(q^*_2 = 0\) and, thus, \(\Pi^*\) is liable to increase in \(\alpha\) and approach the optimal profit \(\Pi^C\) in the commitment scenario. Denote \(\alpha_s := \inf \{\alpha : q^*_2 (\alpha) = 0\}\) corresponding to the boundary between Strategy \(N\overline{N}R_A\) and Strategy \(N\emptyset R_A\). Also, denoting \(\mu^*_n = v_n - c_n (1 + \alpha_s) - h\alpha_s\), we have:

\[
\frac{d\Pi^*_\overline{N}NR_A}{d\alpha}\bigg|_{\alpha \to \alpha_s} = \frac{1}{2\alpha_s (1 + k_r \alpha_s) \left(v_n (1 + k_r \alpha_s) + 2\alpha_s v_r\right)} \times \\
\left\{\mu^*_n [v_n (1 + k_r \alpha_s)^3 (2 - \rho) (\mu^*_n - (\alpha_s c_n + h\alpha_s)) - 4\alpha_s v_r (1 - \rho) \mu^*_n - 2\alpha_s v_r (2 - \rho) (\alpha_s c_n + h\alpha_s)]ight. \\
+ 2\mu^*_n k_r \rho^2 \rho v_r \left[4(v_n - 2\alpha v_r - 3\alpha c_n k_r) + 2\alpha_s (1 + k_r) (\mu^*_n + c_n)\right] \\
- 2\mu^*_n k_r \rho^2 \rho [h v_n (1 + k_r \alpha_s) + 5v_r (v_n - \mu^*_n)] + 2v_r (v_n - c_n) (2 + k_r \alpha_s)\right\}.
\]

(A.14)

For \(k_r > 0\), the right hand side of expression (A.14) is \(> 0\) when

\[v_r > \hat{v}_{r2} := \frac{\sqrt{x^2 + 4k_r \rho^2 \alpha s v_n (1 + k_r \alpha_s) - x}}{4k_r \rho \alpha^2 s},\]

where \(x = 2 \left(1 + k_r \alpha_s\right)^2 \left[(1 - \rho) \mu^*_n + \alpha_s (c_n + h)\right] - \rho \alpha s \left[c_n (1 - k_r) (1 + k_r \alpha_s) + h (1 - k_r \alpha_s)\right] + 2k_r \alpha s \mu^*_n\)

and \(y = (2 - \rho) \left(1 + k_r \alpha_s\right)^2 \left[\mu^*_n - \alpha_s (c_n + h)\right] - 2k_r \rho^2 \rho s\).

For \(k_r = 0\), the right hand side of expression (A.14) is \(> 0\) when

\[v_r > \hat{v}_{r2} := \frac{v_n (2 - \rho) (\mu^*_n - (\alpha_s c_n + h\alpha_s))}{2\alpha s (2 (1 - \rho) \mu^*_n + (2 - \rho) (\alpha_s c_n + h\alpha_s))}.
\]
The result follows by setting \( \hat{v}_r = \max \{ \tilde{v}_{r1}, \tilde{v}_{r2} \} \).

A.5 Proof of Proposition 4 (§5.1)

As shown in the proof of Proposition 3(ii)(b), the firm’s profit increases in the defect rate \( \alpha \) at the boundary between equilibrium strategies \( NNRA \) and \( N\emptyset RA \), where the new product is displaced by the refurbished product in the second period (i.e., \( q_2^r \) becomes equal to 0). We show that the firm’s profit: (i) increases in the perceived quality \( v_r \) of the refurbished product; and, (ii) decreases in the hassle cost \( h \), under both: (a) Strategy \( NNRA \); and, (b) Strategy \( N\emptyset RA \).

Substituting the optimal/equilibrium prices and sales quantities from Tables 2 and 4 (as functions of \( q_1 \)) into the firm’s discounted two-period profit function in (4.5), we obtain:

\[
\Pi_{NNRA}(q_1) = \frac{\rho \mu_n^2}{4v_n} + \left[ \mu_n (1 - \rho) + \frac{\rho \alpha (v_n \mu_r - v_r \mu_n)}{v_n (1 + k_r \alpha)} \right] q_1 - \left[ \frac{v_n (4 - 3\rho)}{4} + \frac{\rho \alpha^2 v_r (v_n - v_r)}{v_n (1 + k_r \alpha)^2} \right] q_1^2
\]

\[
\Pi_{N\emptyset RA}(q_1) = \left[ \mu_n + \frac{\rho \alpha \mu_r}{1 + k_r \alpha} \right] q_1 - \left[ v_n + \frac{\rho \alpha v_r (\alpha + 2 (1 + k_r \alpha))}{(1 + k_r \alpha)^2} \right] q_1^2
\]

(A.15) (A.16)

(i)(a) The firm’s profit increases in \( v_r \) in Strategy \( NNRA \):

In Strategy \( NNRA \), we have \( q_1^r (v_r) = 2 (1 + k_r \alpha) \left( \frac{v_n \mu_n (1 - \rho) (1 + k_r \alpha) + \rho \alpha (v_n \mu_r - v_r \mu_n)}{(v_n (1 + k_r \alpha)^2 (4 - 3\rho) + 4 \rho \alpha^2 v_r (v_n - v_r))} \right) > 0 \) (from Lemma 2(a) in Appendix A.3). Solving for \( v_r : q_1^r (v_r) = 0 \), we obtain:

\[
v_r^{q_1^r} := \frac{v_n \left[ \rho \alpha \left( (1 + k_r \alpha) c_r + h k_r \alpha \right) - \mu_n (1 - \rho) (1 + k_r \alpha) \right]}{\rho \alpha (v_n - \mu_n)}
\]

Note that \( v_r \leq v_r^{q_1^r} \iff q_1^r (v_r) \leq 0 \). However, substituting \( q_1^r (v_r) \) for \( q_1 \) in equation (A.15), we can also show that \( \Pi_{NNRA}^* \) is convex in \( v_r \), with \( v_r \leq v_r^{q_1^r} \iff \frac{d \Pi_{NNRA}^*}{dv_r} \leq 0 \). Since \( q_1^r (v_r) > 0 \), \( v_r > v_r^{q_1^r} \) must hold and, thus, \( \frac{d \Pi_{NNRA}^*}{dv_r} > 0 \).

(i)(b) The firm’s profit increases in \( v_r \) in Strategy \( N\emptyset RA \):

In Strategy \( N\emptyset RA \), we have \( q_{11}^r (v_r) = \max \{ q_{11}^r (v_r), q_{12}^r (v_r) \} > 0 \) (from Lemma 2(a) in Appendix A.3), where:

\[
q_{11}^r (v_r) = \frac{(1 + k_r \alpha) (\mu_n (1 + k_r \alpha) + \rho \alpha \mu_r)}{2 \left( v_n (1 + k_r \alpha)^2 + \rho \alpha v_r (\alpha + 2 (1 + k_r \alpha)) \right)} \quad \text{and} \quad q_{12}^r (v_r) = \frac{\mu_n (1 + k_r \alpha)}{v_n (1 + k_r \alpha) + 2 \alpha v_r}.
\]

Solving for \( v_r : q_{11}^r (v_r) = 0 \), we obtain:

\[
v_r^{q_{11}^r} := \frac{\rho \alpha ((1 + k_r \alpha) c_r + h k_r \alpha) - \mu_n (1 + k_r \alpha)}{\rho \alpha}
\]

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Note that \( v_r \leq v_{q_1}^* \iff q_1^* (v_r) \leq 0 \). However, using equation (A.16), we can also show that \( \Pi^*_{NNR_A} (q_1^* (v_r)) \) is convex in \( v_r \), with \( v_r \leq v_{q_1}^* \iff \frac{d}{dv_r} \Pi^*_{NNR_A} (q_1^* (v_r)) \leq 0 \). Since \( q_1^* (v_r) > 0 \), the following conditions, in turn, must hold in the two possible cases in Strategy \( N \not\emptyset R_A \) (see Table 7):

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: ( q_1^* (v_r) \geq q_{12}^* (v_r) )</td>
<td>( q_1^* (v_r) &gt; 0 )</td>
</tr>
<tr>
<td>Case 2: ( q_{12}^* (v_r) &gt; q_1^* (v_r) )</td>
<td>( q_{12}^* (v_r) &gt; 0 )</td>
</tr>
</tbody>
</table>

Table 7: Conditions that must hold in Strategy \( N \not\emptyset R_A \)

In Case 1, \( v_r > v_{q_1}^* \) must hold for \( q_1^* (v_r) \) to be > 0, implying that \( \frac{d \Pi^*_{NNR_A}}{dv_r} > 0 \). Also, since \( q_{12}^* (v_r) > 0 \Rightarrow q_1^* (v_r) > 0 \), we have that \( v_r > v_{q_1}^* \) holds in Case 2 as well and, therefore, that \( \frac{d \Pi^*_{NNR_A}}{dv_r} > 0 \) again.

Thus, \( \frac{d \Pi^*_{NNR_A}}{dv_r} > 0 \) in Strategy \( N \not\emptyset R_A \).

(ii)(a) **The firm’s profit decreases in \( h \) in Strategy \( NNR_A \):**

Using equation (A.15) and applying the envelope theorem, we have 
\[
\frac{d \Pi^*_{NNR_A}}{dh} = \left. \frac{\partial \Pi^*_{NNR_A}(q_1)}{\partial h} \right|_{q_1=q^*_1}.
\]
Therefore,
\[
\frac{d \Pi^*_{NNR_A}}{dh} = \frac{-2 \alpha q_1 [(1 + k_r \alpha - \rho) v_n - \rho \alpha v_r] + \rho \alpha \mu_n (1 + k_r \alpha)}{2 v_n (1 + k_r \alpha)} \bigg|_{q_1=q^*_1} \tag{A.17}
\]

Since the denominator of the right hand side (RHS) of equation (A.17) is positive, it suffices to show that the numerator term \( N_{h_1} := 2 \alpha q_1 [(1 + k_r \alpha - \rho) v_n - \rho \alpha v_r] + \rho \alpha \mu_n (1 + k_r \alpha) \) is greater than zero. In Strategy \( NNR_A \), we have:
\[
q_2^*(q_1) = \frac{\mu_n (1 + k_r \alpha) - q_1 [v_n (1 + k_r \alpha) + 2 \alpha v_r]}{2 v_n (1 + k_r \alpha)} \tag{A.18}
\]

Denote the numerator on the RHS of equation (A.18) as \( N_{h_2} := \mu_n (1 + k_r \alpha) - q_1 [v_n (1 + k_r \alpha) + 2 \alpha v_r] \).

Note that \( N_{h_2} \geq 0 \), since \( q_2^* \geq 0 \). Also, we have \( N_{h_1} - \rho \alpha N_{h_2} = \alpha q_1 v_n [2 - \rho + (2 + \rho) k_r \alpha] > 0 \), implying that \( N_{h_1} > 0 \) and, therefore, that \( \frac{d \Pi^*_{NNR_A}}{dh} < 0 \).

(ii)(b) **The firm’s profit decreases in \( h \) in Strategy \( N \not\emptyset R_A \):**

In Strategy \( N \not\emptyset R_A \), we have \( q_1^*(h) = \max \{ q_{11}^*(h), q_{12}^*(h) \} > 0 \) (from Lemma 2(a) in Appendix A.3), where:
\[
q_{11}^*(h) = \frac{(1 + k_r \alpha) (\mu_n (1 + k_r \alpha) + \rho \alpha \mu_r)}{2 (v_n (1 + k_r \alpha)^2 + \rho \alpha v_r (1 + k_r \alpha))} \quad \text{and} \quad q_{12}^*(h) = \frac{\mu_n (1 + k_r \alpha)}{v_n (1 + k_r \alpha) + 2 \alpha v_r}.
\]

When \( q_{11}^*(h) \geq q_{12}^*(h) \), substituting \( q_{11}^*(h) \) for \( q_1 \) in equation (A.16) and differentiating with respect to \( h \), we obtain:
\[
\frac{d}{dh} \Pi^*_N (q_{11}^*(h)) = -\frac{\alpha q_{11}^*(h) [1 + k_r \alpha (1 + \rho)]}{1 + k_r \alpha} < 0.
\]
When $q_{12}^*(h) > q_{11}^*(h)$, substituting $q_{12}^*(h)$ for $q_1$ in equation (A.16), we can show that $\Pi_{N\emptyset R_A}^*(q_{12}^*(h))$ is convex in $h$, with $h \leq h^\Pi \iff \frac{d}{dh} \Pi_{N\emptyset R_A}^*(q_{12}^*(h)) \leq 0$, where $h^\Pi = h : \frac{d}{dh} \Pi_{N\emptyset R_A}^*(q_{12}^*(h)) = 0$ (we omit the expression of $h^\Pi$ for brevity). However, we can also show that $h \geq h^\Pi \implies q_{12}^*(h) < 0$. Since $q_1^* > 0$, $h < h^\Pi$ must hold and, thus, $\frac{d}{dh} \Pi_{N\emptyset R_A}^*(q_{12}^*(h)) < 0$. \qed

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